VALUES OF LAND, GOLD, RENEWABLE RESOURCES AND CAPITAL IN A GROWTH MODEL WITH AGRICULTURAL AND INDUSTRIAL SECTORS

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Abstract. *This paper deals with a dynamic interdependence between values of gold, capital, land and renewable resource in a three sector growth model with endogenous wealth and renewable resources. The model is a synthesis of the neoclassical growth theory, Ricardian theory and growth theory with renewable resources. The economic system consists of the households, industrial, agricultural, and resource sectors and given land and gold. The economic system is perfectly competitive. We build the model of capital and renewable resources with portfolio equilibrium. We provide a computational procedure for simulating the model. The simulated case has a unique stable equilibrium point. We plot the motion of the dynamic system. We also conduct comparative dynamic analysis with regard to changes in the propensity to use gold, the propensity to consume resources, the propensity to consume housing, the propensity to consume agricultural goods, the propensity to consume industrial goods, the propensity to hold wealth, and the population.*

Keywords: land value; price of gold; economic growth; economic structure; stock of renewable resources

1. Introduction

This paper is concerned an important but seldom addressed issue in modern economics: dynamics of economic growth and portfolio equilibrium. Land/housing values, precious metals, and other different forms of assets are important parts of economic systems. Households of contemporary economies hold different kinds of assets such as housing, land, stocks, precious metals, gold, cashes in different currencies. Although modern economics has many dynamic models, there are only a few growth models built on proper microeconomic foundation which take account of portfolio equilibrium between gold, land and physical wealth.

The purpose of this study is to develop a dynamic growth model with capital accumulation and renewable resource change. We take account of portfolio equilibrium dynamics in association with economic growth and structural changes. The history of economic analysis shows that it is difficult to build genuine dynamic models with interactions between multiple kinds of capitals on microeconomic foundation. This study makes a contribution to the literature of economic dynamics by developing a dynamic interdependence between gold, capital, land and resource values in an economy with industrial,

industrial, and resource sectors. The model treats wealth and renewable resources as endogenous.

The model is based on the neoclassical growth theory, Ricardian theory and growth theory with renewable resources. The modelling of agricultural and industrial sectors is based on the neoclassical growth theory (Solow, 1956; Uzawa, 1961; Stiglitz, 1967; Drugeon and Venditti, 2001; Erceg et al. 2005). Nevertheless, only a few efforts are made to introduce portfolio equilibrium between different assets into the neoclassical economic growth theory. This study introduces dynamics of renewable resources into the neoclassical growth theory. Stock of renewable resources is changeable according how fast agents utilize resources and how fast renewable resources grow.

Natural resources are incorporated into the neoclassical growth theory in the 1970s (e.g., Plourde, 1970, 1971; Stiglitz, 1974; Clark, 1976; Dasgupta and Heal, 1979). Gordon (1956) emphasized the need for a dynamic approach to fisheries economics: "The conservation problem is essentially one which requires a dynamic formulation… The economic justification of conservation is the same as that of any capital investment – by postponing utilization we hope to increase the quantity available for use at a future date. In the fishing industry we may allow our fish to grow and to reproduce so that the stock at a future date will be greater than it would be if we attempted to catch as much as possible at the present time. … [I]t is necessary to arrive at an optimum which is a catch per unit of time, and one must reach this objective through consideration of the interaction between the rate of catch, the dynamics of fish population, and the economic time-preference schedule of the community or the interest rate on invested capital.

This is a very complicated problem and I suspect that we will have to look to the mathematical economists for assistance in clarifying it." Munro and Scott (1985) explained that in the 1950s it was quite difficult to develop workable dynamic models of resources. Solow (1999) also argues for the necessity of taking account of natural resources in the neoclassical growth theory. Solow held that if the resource good is used as one of the inputs in the production, then it is easy to incorporate the use of renewable resources into the neoclassical growth model. But he did not show how to incorporate possible consumption of renewable resource into the growth model. There are only a few models of growth and renewable resources which treat the renewable resource as a source of utility (see, Beltratti, et al., 1994, Ayong Le Kama, 2001; Eliasson and Turnovsky, 2004; Alvarez-Cuadrado and van Long, 2011). Our model contains the renewable resource as a source of utility.

Gaffney (2008: 119) pointed out that "Most economists today live in a two-factor world: There is just labor and capital. Land, so central to classical political economy, has been swallowed into capital and "disappeared."" We introduce economic mechanisms to determine land values. Land use is a central concern of classical economics. Ricardo (1821: preface) pointed out: "The produce … is divided among three classes of the commodity, namely, the proprietor of land, the owners of the stock or capital necessary for its cultivation, and laborers by whose industry it is cultivated. But in different stages of the society, the proportions of the whole produce of the earth which will be allotted to each of these classes, under the names of rent, profits, and wages, will be essentially different; depending mainly on the actual fertility of the soil, on the accumulation of capital and population, and on the skill, ingenuity, and the instruments in agriculture."

There are many studies on the Ricardian economic system (Barkai, 1959, 1966; Pasinetti, 1960, 1974; Cochrane, 1970; Brems, 1970; Caravale and Tosato, 1980; Casarosa, 1985; Negish, 1989; Morishima, 1989). Nevertheless, we can use what Ricardo (1821: preface) observed long time ago to describe the contemporary situation: "To determine the laws which regulate this distribution, is the principal problem in Political Economy: much as the science has been improved by the writings of Turgot, Stuart, Smith, Say, Sismondi, and others, they afford very little satisfactory information respecting the natural course of rent, profit, and wages." In Ricardo's statement there is no reference to land value (price).

The traditional Ricardian theory does not determine land price dynamics. Nevertheless, there are many empirical studies on housing and land prices (e.g., Bryan and Colwell, 1982; Case and Quigley, 1991; Chinloy, 1992; Clapp and Giaccotto, 1994; Calhoun, 1995; Quigley, 1995; Capozza and Seguin, 1996; Cho, 1996; Alpanda, 2012; Alexander, 2013; Du and Peiser, 2014; Kok et al. 2014). Deaton and Laroque (2001) observe that "The user cost of land reduces the resources available for consumption of reproducible goods, so that the introduction of intrinsically valuable land into a growth model lowers the equilibrium stock of capital and raises the equilibrium interest rate. On the asset side, the presence of land causes life-cycle savings to be reallocated away from productive capital towards land. … Land markets, far from generating saving and growth, are inimical to capital formation." By including land and its value as endogenous variable the new equilibrium is typically obtained with a lower stock of capital and a higher rate of interest. This is the effect identified by Nichols (1970), Feldstein (1977), Drazen and Eckstein (1988), Jappelli and Pagano (1994), and which was first explored by Allais (1948).

This study does not only examine wealth and land value, but also introduces gold value into the neoclassical growth theory with portfolio equilibrium between land, gold and physical wealth. We examine gold price dynamics within a neoclassical growth model with agricultural and industrial sectors. Dynamics of gold prices are seldom properly study in the literature of economic dynamics (Barro, 1979; Bordo and Ellson, 1985; Dowd and Sampson, 1993; Chappell and Dowd, 1997). It should be noted that this study is a synthesis of Zhang's recent two models (Zhang, 2011, 2016).

This paper is organized as follows. Section 2 develops the growth model of endogenous physical and renewable resource dynamics with land distribution and housing. Section 3 examines dynamic properties of the model and simulates the model. Section 4 carries out comparative dynamic analysis with regard to changes in the propensity to use gold, the propensity to consume resources, the propensity to consume housing, the propensity to consume industrial goods, the propensity to consume agricultural goods, the propensity to save, and the total factor productivity of the industrial sector. Section 5 concludes the study. The appendix proves the results in section 3.

2. The model

The model is based on Zhang's two models. Zhang (2011) builds a growth model with renewable resources and capital accumulation, while Zhang (2016) constructs a growth model of portfolio equilibrium choice between land, gold, and physical wealth. This study deals with a dynamic economy with industrial, agricultural and renewable resource sectors by integrating the two models. Most aspects of the production sectors are similar to the standard one-sector growth model in the neoclassical growth theory (Burmeister and Dobell, 1970; Barro and Sala-i-Martin, 1995).

Production sectors use physical capital, labor and land inputs. Exchanges take place in perfectly competitive markets. Factor markets work well; factors are inelastically supplied and the available factors are fully utilized at every moment. Saving is undertaken only by households. We select the commodity produced by the industrial sector to serve as numeraire. All the other prices are measured relative to its price. The industrial production is the same as that in Solow's one-sector neoclassical growth model. It is a commodity used both for investment and consumption. The agricultural sector produces agricultural goods, used for consumption. The population N is homogenous and constant. The total land L is owned by the population and is distributed between housing and agricultural production in free land market. The rate of interest $r(t)$ and wage rate $w(t)$ are determined in competitive markets.

The industrial sector

The industrial sector combines labor force $N_i(t)$ and physical capital $K_i(t)$ to produce output $F_i(t)$ in the following way

$$F_i(t) = A_i K_i^{\alpha_i}(t) N_i^{\beta_i}(t), \ \ A_i, \alpha_i, \beta_i > 0, \ \ \alpha_i + \beta_i = 1, \tag{1}$$

where A_i, α_i and β_i are parameters. The marginal conditions are given by

$$r(t) + \delta_k = \frac{\alpha_i F_i(t)}{K_i(t)}, \ \ w(t) = \frac{\beta_i F_i(t)}{N_i(t)}, \tag{2}$$

where δ_k is the fixed depreciation rate of physical capital.

The agricultural sector

The agricultural sector use capital $K_a(t)$, labor force $N_a(t)$, and land $L_a(t)$ to produce output $F_a(t)$ in the following way

$$F_a(t) = A_a K_a^{\alpha_a}(t) N_a^{\beta_a}(t) L_a^{\varsigma}(t), \ \ A_a, \alpha_a, \beta_a, \varsigma > 0, \ \ \alpha_a + \beta_a + \varsigma = 1, \tag{3}$$

where A_a, α_a, β_a, and ς are parameters. The marginal conditions are

$$r(t) + \delta_k = \frac{\alpha_a \, p_a(t) F_a(t)}{K_a(t)}, \ \ w(t) = \frac{\beta_a \, p_a(t) F_a(t)}{N_a(t)}, \ \ R_L(t) = \frac{\varsigma \, p_a(t) F_a(t)}{L_a(t)}, \tag{4}$$

where $p_a(t)$ is the price of agricultural goods and $R_L(t)$ is the land rent.

Change of renewable resources

We model resources dynamics on the basis of Zhang (2011). We use $X(t)$ to represent the stock of the resource. We use the logistic model to describe growth of renewable resources (e.g., Brander and Taylor, 1997; Brown, 2000; Hannesson, 2000; Cairns and Tian, 2010; and Farmer and Bednar-Friedl, 2011). The natural growth rate of the resource is given by a logistic function as follows

$$\phi_0 \, X(t) \left(1 - \frac{X(t)}{\phi(L_x(t))} \right),$$

where the variable, $\phi(L_x(t))$, is the maximum possible size for the resource stock, called the carrying capacity of the resource which is dependent on the land $L_x(t)$ used for renewable resource. and the variable, ϕ_0, is "uncongested" or "intrinsic" growth rate of the renewable resource. If the stock is equal to ϕ, then the growth rate should equal zero. If the carrying capacity is much larger than the current stock, then the growth rate per unit of the stock is approximately equal to the intrinsic growth rate. It should be noted that there are some alternative approaches to renewable resources (Levhari and Withagen, 1992; Tornell and Velasco, 1992; Benchekroun, 2003; Long and Wang, 2009; Fujiwara, 2011). We $F_x(t)$ stand for the harvest rate of the resource. The change rate in the stock is then equal to the natural growth rate minus the harvest rate

$$\dot{X}(t) = \phi_0 \, X(t) \left(1 - \frac{X(t)}{\phi(L_x)} \right) - F_x(t). \tag{5}$$

As in Gordon, (1954), renewable resource is nationally owned and open-access (see Alvarez-Guadrado and VonLong, 2011). With open access, harvesting occurs up to the point at which the current return to a representative entrant equals the entrant's cost. The resource sector employs labor force $N_x(t)$ and capital stock $K_x(t)$ to the following harvesting production function

$$F_x(t) = A_x X^b(t) L_x^{b_x}(t) K_x^{\alpha_x}(t) N_x^{\beta_x}(t), \ \ A_x, x, b, b_x, \alpha_x, \beta_x > 0, \ \ \alpha_x + \beta_x = 1, \tag{6}$$

where $A_x, m_x, b, b_x, \alpha_x$ and β_x are parameters. The Schaefer harvesting production function (Schaefer, 1957)

$$F_x(t) = A_x X(t) N_x(t),$$

is a special case of (6). The function with fixed capital and technology is widely applied to fishing (see also, Paterson and Wilen, 1977; Milner-Gulland and Leader-Williams, 1992; Bulter and van Kooten, 1999). We use $p_x(t)$ to denote the price of the resource. The marginal conditions are

$$r(t) + \delta_k = \frac{\alpha_x p_x(t) F_x(t)}{K_x(t)}, \quad w(t) = \frac{\beta_x p_x(t) F_x(t)}{N_x(t)}. \tag{7}$$

Choice between physical wealth, gold, and land

We use $p_L(t)$ and $p_G(t)$ to represent, respectively, the prices of land and gold. It is assumed that gold can be "rented" through markets for decoration use. The rent of gold is denoted by $R_G(t)$. The gold owned by the representative household is assumed to be fully used either by the household for decoration or rented out to other households. Consider now an investor with one unity of money. He can either invest in capital good thereby earning a profit equal to the net own-rate of return $r(t)$ or invest in land (gold) thereby earning a profit equal to the net own-rate of return $R_L(t)/p_L(t)$ $(R_G(t)/p_G(t))$. As we assume capital, gold and land markets to be at competitive equilibrium at any point in time, three options yield equal returns, i.e.

$$\frac{R_G(t)}{p_G(t)} = \frac{R_L(t)}{p_L(t)} = r(t). \tag{8}$$

Obviously equations (8) are established under many strict conditions. For instance, we omit any transaction costs and any time delay for buying and selling. Expectations on gold and land are complicated. Equation (8) also implies perfect information.

The current income and disposable income

We use lot size to stand for housing. The representative household decides consumption levels of industrial good, agricultural good, renewable resource, gold and lot size, as well as on how much to save. This study uses the approach to consumers' behavior proposed by Zhang (1993, 2005). We use $\bar{k}(t)$ to stand for the representative household's physical wealth, $\bar{g}(t)$ for the amount of gold owned by the household, and $\bar{l}(t)$ for the lot size. The total value of wealth owned by the household $a(t)$ is the sum of the three assets' values

$$a(t) = \bar{k}(t) + p_L(t)\bar{l}(t) + p_G(t)\bar{g}(t). \tag{9}$$

The representative household's current income is the sum of the interest payment of physical wealth $r(t)\bar{k}(t)$, the wage payment $w(t)$, the gold interest income $R_G(t)\bar{g}(t)$ and the land revenue $R_L(t)\bar{l}(t)$ as follows

$$y(t) = r(t)\bar{k}(t) + w(t) + R_L(t)\bar{l}(t) + R_G(t)\bar{g}(t). \tag{10}$$

The household's disposable income is the sum of the current income and value of the household's assets, i.e.

$$\hat{y}(t) = y(t) + a(t). \tag{11}$$

The budget, utility and optimal decision

The representative household would distribute the total available budget between saving $s(t)$, consumption of the commodity $c_i(t)$, consumption of the resource good $c_x(t)$, consumption of the agricultural good $c_a(t)$, gold use $\hat{g}(t)$, and housing, $l_h(t)$. The budget constraint is given by

$$c_i(t) + s(t) + p_a(t)c_a(t) + p_x(t)c_x(t) + R_G(t)\hat{g}(t) + R_L(t)l_h(t) = \hat{y}(t). \tag{12}$$

The representative household has six variables, $s(t)$, $c_i(t)$, $c_x(t)$, $c_a(t)$, $\hat{g}(t)$, and $l_h(t)$ to decide. We assume that the household has the following utility function

$$U(t) = c_i^{\xi_0}(t)c_a^{\mu_0}(t)c_x^{\chi_0}(t)l_h^{\eta_0}(t)\hat{g}^{\gamma_0}(t)s^{\lambda_0}(t), \ \ \xi_0, \mu_0, \chi_0, \eta_0, \gamma_0, \lambda_0 > 0,$$

in which $\xi_0, \mu_0, \chi_0, \eta_0, \gamma_0$ and λ_0 are the household's elasticities of utility with regard to the commodity, the agricultural goods, the resource, housing, use of gold, and saving. We call $\xi_0, \mu_0, \chi_0, \eta_0$, γ_0 and λ_0, respectively, propensities to consume the commodity, the agricultural goods, the resource, and housing, the propensity to use gold, and the propensity to hold wealth. Maximizing $U(t)$ subject to (12) yields

$$\begin{aligned} & c_i(t) = \xi\hat{y}(t), \ \ p_a(t)c_a(t) = \mu\hat{y}(t), \ \ p_x(t)c_x(t) = \chi\hat{y}(t), \ \ R_L(t)l_h(t) = \eta\hat{y}(t), \\ & R_G(t)\hat{g}(t) = \gamma\hat{y}(t), \ \ s(t) = \lambda\hat{y}(t), \end{aligned} \tag{13}$$

where

$$\xi \equiv \rho\xi_0, \ \ \mu \equiv \rho\mu_0, \ \ \chi \equiv \rho\chi_0, \ \ \eta \equiv \rho\eta_0, \ \ \gamma \equiv \rho\gamma_0, \ \ \lambda \equiv \rho\lambda_0,$$
$$\rho \equiv \frac{1}{\xi_0 + \mu_0 + \chi_0 + \eta_0 + \gamma_0 + \lambda_0}.$$

Wealth accumulation

According to the definition of $s(t)$, the change in the household's wealth follows the differential following equation

$$\dot{a}(t) = s(t) - a(t). \tag{14}$$

The equation simply states that the change in wealth is equal to the saving minus the dissaving.

Balances of demand and supply for industrial goods

The demand and supply for the industrial sector's output balance at any point in time

$$\dot{K}(t) = F_i(t) - c_i(t)N - \delta_k K(t). \tag{15}$$

According to Say's law, we can consider this equation redundant in the general equilibrium system.

Balances of demand and supply for agricultural goods

The demand and supply for the agricultural sector's output balance at any point in time

$$C_a(t) = c_a(t)N = F_a(t). \tag{16}$$

Balances of demand and supply for renewable resources

The demand and supply for the resource balance at any point of time

$$c_x(t)N = F_x(t). \tag{17}$$

All the land owned by the population

The land owned by the population is equal to the national available land

$$\bar{l}(t)N = L. \tag{18}$$

All the gold owned by the population

The gold owned by the population is equal to the available amounts of gold

$$\bar{g}(t)\bar{N} = G. \tag{19}$$

Gold being fully used for decoration

We neglect possible holdings of gold by the government. The amount of gold used for decoration by the population is equal to the total gold

$$\tilde{g}(t)N = G. \tag{20}$$

Full employment of capital

We use $K(t)$ to stand for the total capital stock. We assume that the capital stock is fully employed. We have

$$K_i(t) + K_a(t) + K_x(t) = K(t). \quad (21)$$

The value of physical wealth and capital

The value of physical capital is equal to the value of physical wealth

$$\bar{k}(t)N = K(t). \quad (22)$$

Full employment of labor force

We assume that labor force is fully employed

$$N_i(t) + N_a(t) + N_x(t) = N. \quad (23)$$

The land market clearing condition

The land is fully used

$$l_h(t)N + L_a(t) + L_x(t) = L. \quad (24)$$

Land use for renewable resources

The land use for residents and agricultural product are determined respectively by the marginal conditions for the household and the agricultural sector. We now introduce a mechanism to decide the amount of land used for renewable resource. The land of renewable resource is assumed to be

$$L_x(t) = \varphi L_a(t). \quad (25)$$

where φ is a constant parameter. This assumption is accepted mainly for convenience of analysis.

We thus built the model. The model describes dynamics of the economic structure and values of land and renewable resources. The model is structurally general as that some well-known models in economic theory can be considered as special cases of this model. The rest of this paper examines dynamic properties of the model.

3. The dynamics and the motion by simulation

The economic system contains many nonlinear relations. It is difficult to analytically explore the properties of the nonlinear dynamic system. For illustration, we simulate the model. The appendix shows that the dynamics of the national economy can be expressed as two differential equations. First, we introduce a variable $z(t)$ by

$$z(t) \equiv \frac{r(t) + \delta_k}{w(t)}.$$

The following lemma provides a computational procedure to plot the motion of the dynamic system. The procedure enables us to know the behavior of the system at any point in time.

Lemma

The motion of the system is determined by the following two differential equations

$$\dot{z}(t) = \Lambda(z(t)),$$
$$\dot{X}(t) = \Omega(z(t),\ X(t)), \tag{26}$$

where the right-hand sides of (25) are functions of $z(t)$ and $X(t)$ determined in the appendix. Moreover, all the other variables are determined as functions of $z(t)$ and $X(t)$ at any point in time by the following procedure: $r(t)$ and $w(t)$ by (A2) $\rightarrow \bar{k}(t)$ by (A26) $\rightarrow K_a(t)$ by (A19) $\rightarrow K_i(t)$ and $K_x(t)$ by (A22) $\rightarrow$ $N_i(t),\ N_x(t)$ and $N_a(t)$ by (A1) $\rightarrow \bar{y}(t)$ by (A14) $\rightarrow$ $\bar{l}$ by (18) $\rightarrow$ $\bar{g} = g$ by (19) and (20) $\rightarrow$ $R_L(t)$ by (A15) $\rightarrow p_L(t),\ R_G(t),$ and $p_G(t)$ by (A24) $\rightarrow a(t)$ by (A27) $\rightarrow$ L_a, L_x and l_h by (A10) $\rightarrow p_a(t)$ by (A5) $\rightarrow F_i(t)$ by (1) $\rightarrow F_a(t)$ by (3) $\rightarrow F_x(t)$ by (6) $\rightarrow p_x(t)$ by (7) $\rightarrow c_i(t),\ c_a(t),\ c_x(t),$ and $s(t)$ by (13).

The lemma shows how to determine the values of $z(t)$ and $X(t)$ and how to determine all the variables in the economic system. The lemma is important as it gives a procedure to follow the motion of the system with computer. As the expressions of the analytical results are tedious, for illustration we specify the parameter values and simulate the model. We specify the parameters as follows

$$N = 5,\ L = 10,\ G = 0.5,\ \alpha_i = 0.3,\ \alpha_a = 0.1,\ \beta_a = 0.2,\ \alpha_x = 0.34,\ A_i = 1,\ A_a = 0.5,$$
$$A_x = 0.5,\ \lambda_0 = 0.5,\ \xi_0 = 0.07,\ \chi_0 = 0.02,\ \eta_0 = 0.01,\ \gamma_0 = 0.02,\ \mu_0 = 0.02,$$
$$\varphi = 1.6,\ \phi = 4,\ \phi_0 = 5,\ b = 0.7,\ b_x = 0.01,\ \delta_k = 0.05. \tag{27}$$

The population is fixed at 5, the land is 10, and gold is 0.5. We assume that the propensity to save is much higher than the propensity to consume industrial goods, resources, and agricultural goods. As shown in the appendix, the following variables are invariant in time

$$l_h = 0.43,\ L_a = 3.02,\ L_x = 4.83,\ \bar{l} = 2,\ \bar{g} = g = 0.1.$$

The initial conditions are specified as follows

$$X(0) = 3.4,\ z(0) = 0.22.$$

The motion of the variables is plotted in Figure 1 where the national gross product (GDP) is

$$Y(t) = F_i(t) + p_a(t)F_a(t) + p_x(t)F_x(t) + l_h N R_L(t).$$

The GDP and national capital stock fall over time till they become stationary. The stock of resources rises. The wage rate, price of land, price of resource, price of agricultural goods, and land rent are reduced, and the rate of interest is enhanced. The output level of the agricultural sector is increased and the output level of the industrial sector is reduced. Some of the force is shifted from the industrial sector to the agricultural sector. The capital inputs of the two sectors are increased. The physical wealth, total wealth, and consumption levels of the two goods are increased.

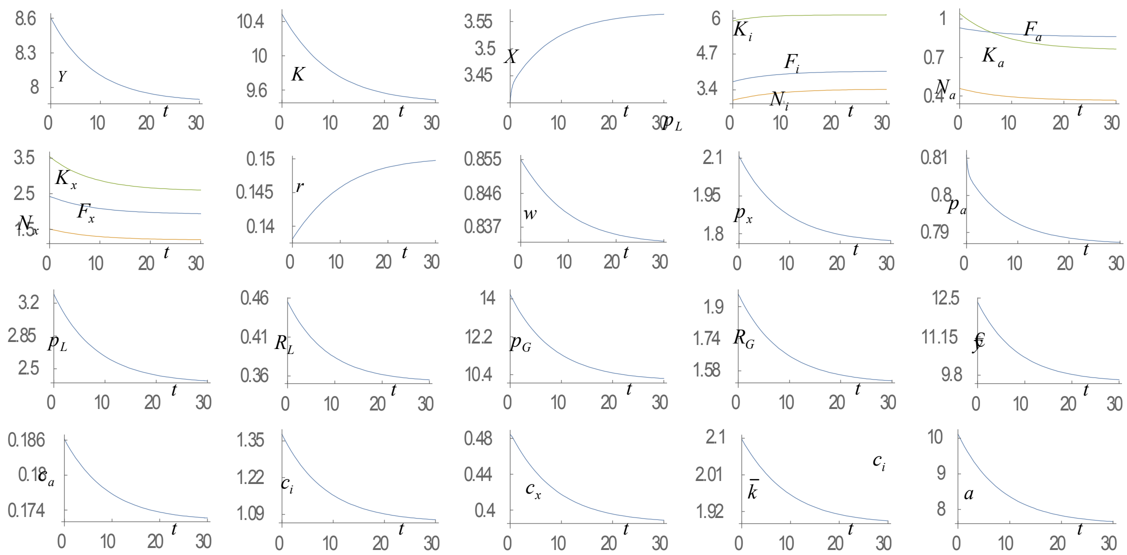

Fig. 1. The Motion of the Economic System

From Fig. 1 we observe that all the variables tend to become stationary in the long term. This implies the existence of some equilibrium point. We confirm the existence of a unique equilibrium point as follows

$$Y = 7.87,\ K = 9.45,\ X = 3.57,\ w = 0.83,\ p_L = 2.34,\ p_G = 10.1,\ R_L = 0.35,$$
$$R_G = 1.52,\ r = 0.15,\ p_a = 1.76,\ p_x = 0.79,\ F_a = 0.86,\ F_i = 4.08,\ F_x = 1.93,$$
$$K_a = 0.76,\ K_i = 6.12,\ K_x = 2.58,\ N_a = 0.36,\ N_i = 3.43,\ N_x = 1.2,\ L_a = 3.02,\ L_x = 4.83,$$
$$\bar{l} = 2,\ l_h = 0.43,\ \bar{g} = g = 0.1,\ \bar{k} = 1.89,\ c_a = 0.17,\ c_x = 0.39,\ c_i = 1.06,\ a = 7.58. \quad (28)$$

The eigenvalues at the equilibrium point are

$$-4.298,\ -0.116.$$

This guarantees the stability of the steady state. This result is important as it guarantees the relevance of comparative dynamic analysis in the next section.

4. Comparative dynamic analysis

We now examine effects of changes in some parameters on the motion of the economic system. We follow the computational procedure in the lemma of the previous section to calibrate the motion of all the variables. The rest of this study uses $\overline{\Delta} x_j(t)$ to stand for the change rate of the variable, $x_j(t)$, in percentage due to changes in a parameter value.

A rise in the propensity to use gold

We first examine the effects of the following change in the propensity to consume resources: $\gamma_0 : 0.01 \Rightarrow 0.011$. The preference change has no impact on the land distribution and gold amount per household. That is, $\overline{\Delta} L_a = \overline{\Delta} L_x = \overline{\Delta} l_h = \overline{\Delta} \bar{l} = \overline{\Delta} \bar{g}$. The effects on the other variables are plotted in Figure 2. As the household's preference for using gold for decoration is increased, both the gold rent and gold value are augmented. The land rent and price rise initially and rise in the long term. The consumption levels of agricultural product, industrial good, natural resource, and physical and total wealth levels rise initially and fall in the long term. The wage rate falls. The price of agricultural product rises initially and falls in the long term. The rate of interest and the price of natural resource are enhanced. The national product and national capital rise initially and fall in the long term. The renewable resource stock falls initially and rises in the long term. The labor force is shifted from the industrial sector to the agricultural and resource sectors. The capital input and output of the industrial sector fall. The capital stocks and output levels of the other two sectors rise initially and fall in the long term.

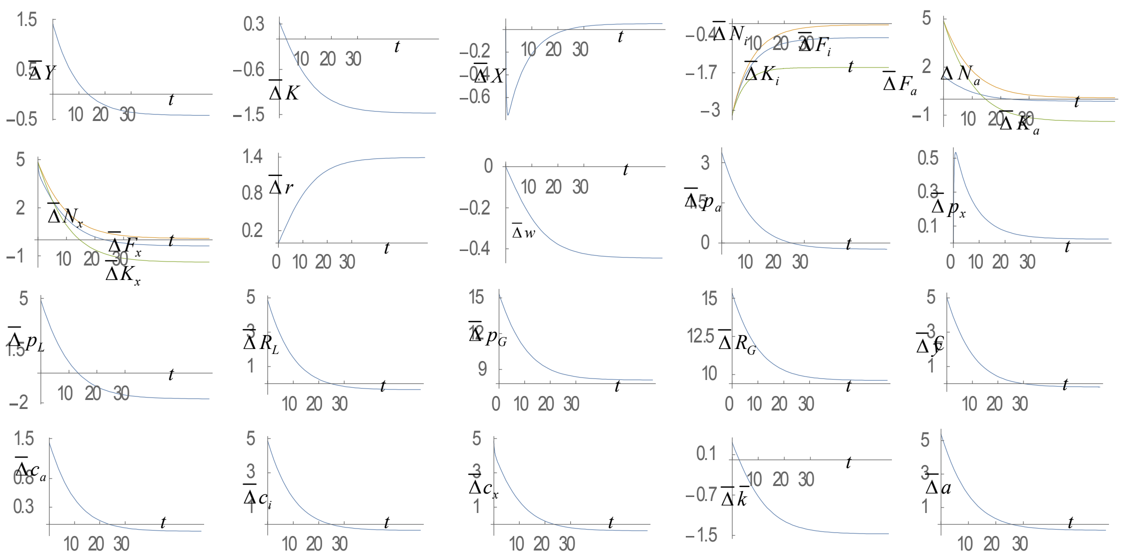

Fig. 2. A Rise in the Propensity to Use Gold

The propensity to consume resources being enhanced

We now allow the propensity to consume resources to change as follows: $\chi_0 : 0.02 \Rightarrow 0.03$. The time-independent variables are not affected. The effects on the other variables are plotted in Figure 3. The household consumes more resources. The stock of resources falls and the resource price is enhanced. The price of agricultural product and wage rate fall. The rate of interest is augmented. The gold rent and value are reduced. The national output is enhanced. The total capital rises initially and falls in the long term. The inputs and output of the resource sector are increased. The inputs and output levels of the other two sectors are reduced. The total wealth and consumption levels of agricultural and industrial goods are reduced.

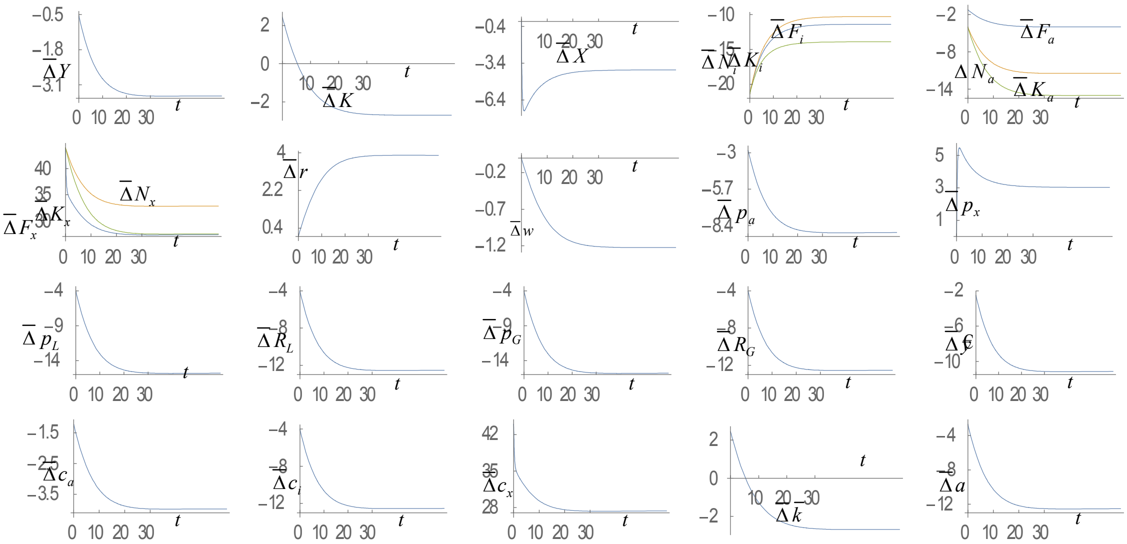

Fig. 3. The Propensity to Consume Resources Being Enhanced

The propensity to consume housing being enhanced

We now examine the impact of the following change in the propensity to consume housing: $\eta_0 : 0.01 \Rightarrow 0.015$. The land use is re-allocated as follow

$$\bar{\Delta} L_a = \bar{\Delta} L_x = -9.73, \ \bar{\Delta} l_h = 35.4, \ \bar{\Delta} \bar{l} = 0, \ \bar{\Delta} \bar{g} = 0.$$

The household has larger house size. The resource and agricultural sectors use less land. The effects on the other variables are plotted in Figure 4. The land rent and value are increased. The gold rent and value rise initially and fall in the long term. The national output rises. The national capital rises initially and falls in the long term. The price of agricultural goods and price of resources are increased. The wage rate is lowered in tandem with rising in the rate of interest.

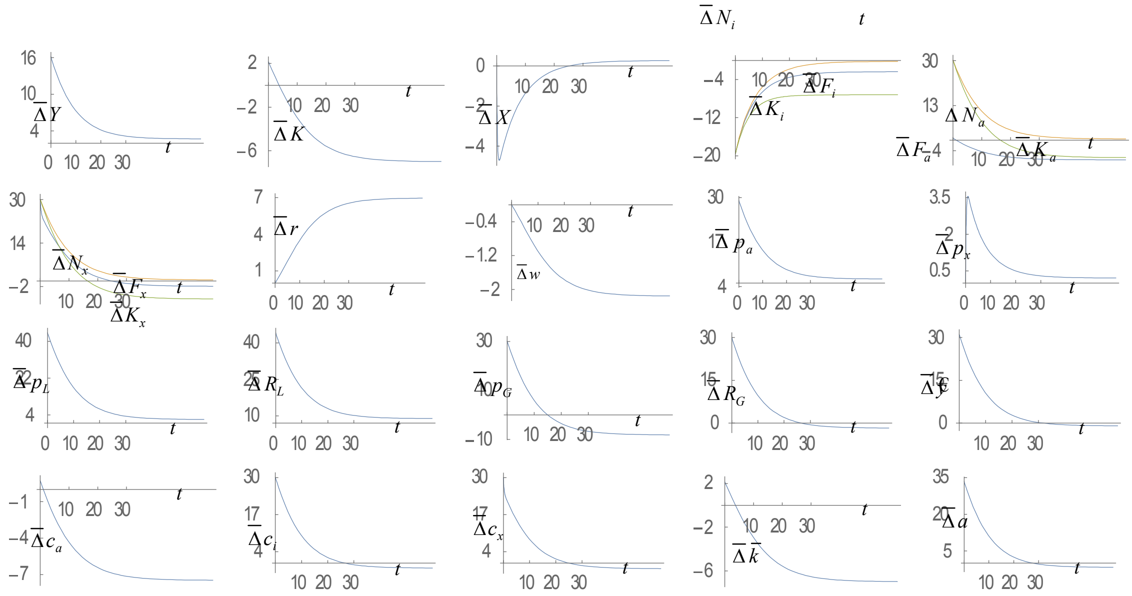

Fig. 4. The Propensity to Consume Housing Being Enhanced

The propensity to consume industrial goods being enhanced

We now allow the propensity to consume industrial goods as follows: $\xi_0 : 0.07 \Rightarrow 0.073$. The land use pattern and gold amount per household are not affected. The effects on the other variables are plotted in Figure 5. The total capital stock and the GDP are lowered. The representative household owns less physical wealth and wealth. The household consumes more industrial goods and less resources and agricultural goods. The stock of resource is increased in association with falling price of the resource. The wage rate is reduced and the rate of interest is enhanced. The values and rents of land and gold are reduced. The price of agricultural product is lowered. The output levels of the agricultural and resource sectors are reduced. The output level of the industrial sector is increased.

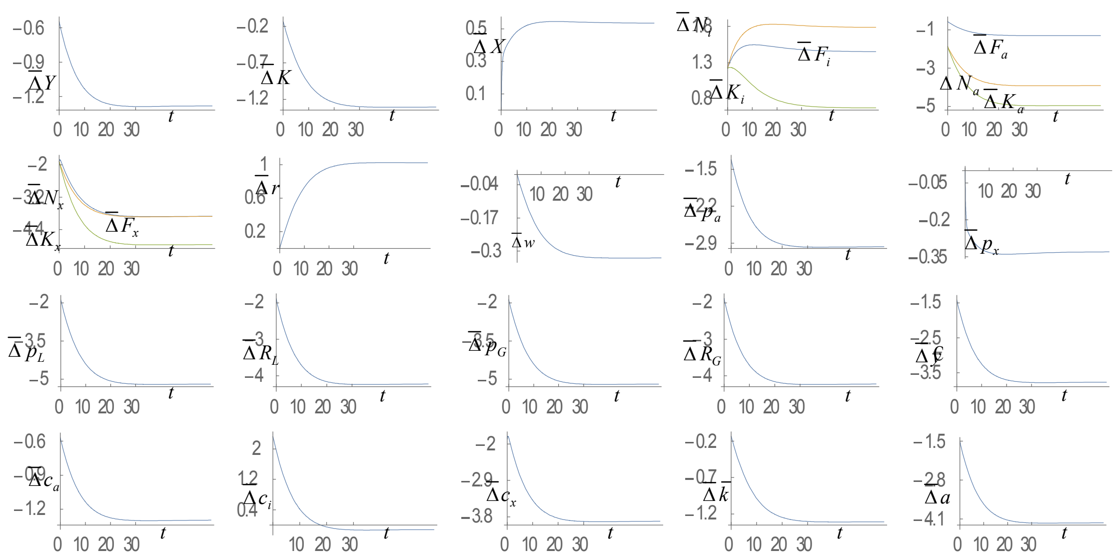

Fig. 5. The Propensity to Consume Industrial Goods Being Enhanced

The propensity to consume agricultural goods being increased

We now allow the propensity to consume agricultural goods to be increased as follows: $\mu_0 : 0.02 \Rightarrow 0.023$. The impact on land use pattern is as follows

$$\overline{\Delta} L_a = \overline{\Delta} L_x = 2.89, \; \overline{\Delta} l_h = -10.53, \; \overline{\Delta} \bar{l} = \overline{\Delta} \bar{g} = 0.$$

More land is devoted to agricultural and resource supplies and less to housing. The effects on the other variables are plotted in Figure 6. The household augments the consumption levels of agricultural product, industrial good, and resource. The household has more total wealth. The household's physical wealth rises initially and falls in the long term. The wage rate falls and rate of interest rises. The prices and rents of land and gold are augmented. The national output rises and the resource stock falls. The national capital rises initially and falls in the long term.

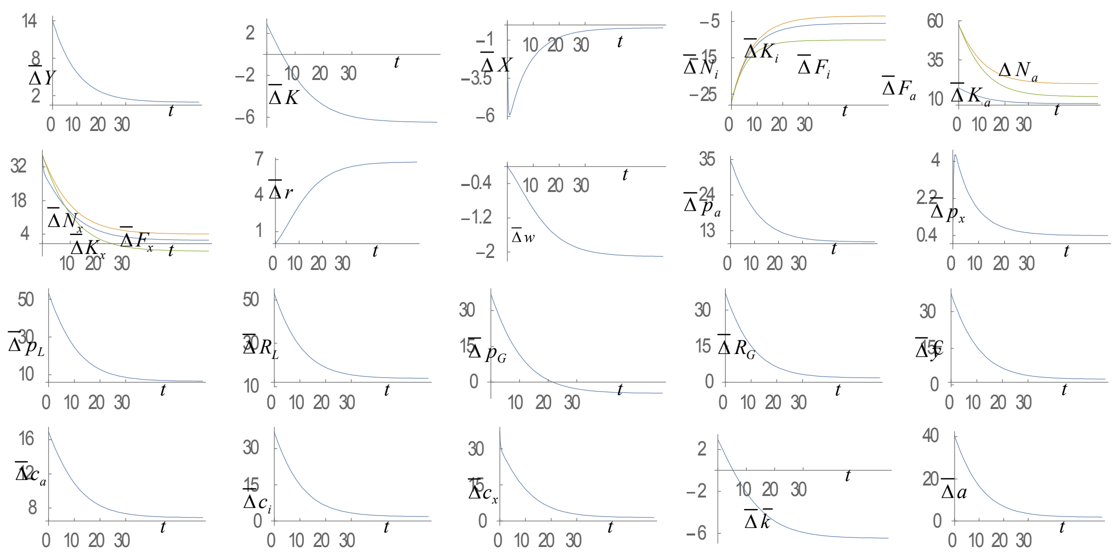

Fig. 6. The Propensity to Consume Agricultural Goods Being Increased

The propensity to save being enhanced

We now allow the propensity to save to rise as follows: $\lambda_0 : 0.5 \Rightarrow 0.51$. The land use allocation is not changed. The effects on the other variables are plotted in Figure 7. The physical and total wealth fall initially and rise in the long term. The consumption levels of industrial goods, agricultural goods, and resource fall initially and rise in the long term. The resource stock rises initially and falls in the long term. The national output and national capital fall initially and rise in the long term. The wage rate rises and the rate of interest falls. The prices and rents of gold and land fall initially and rise in the long term.

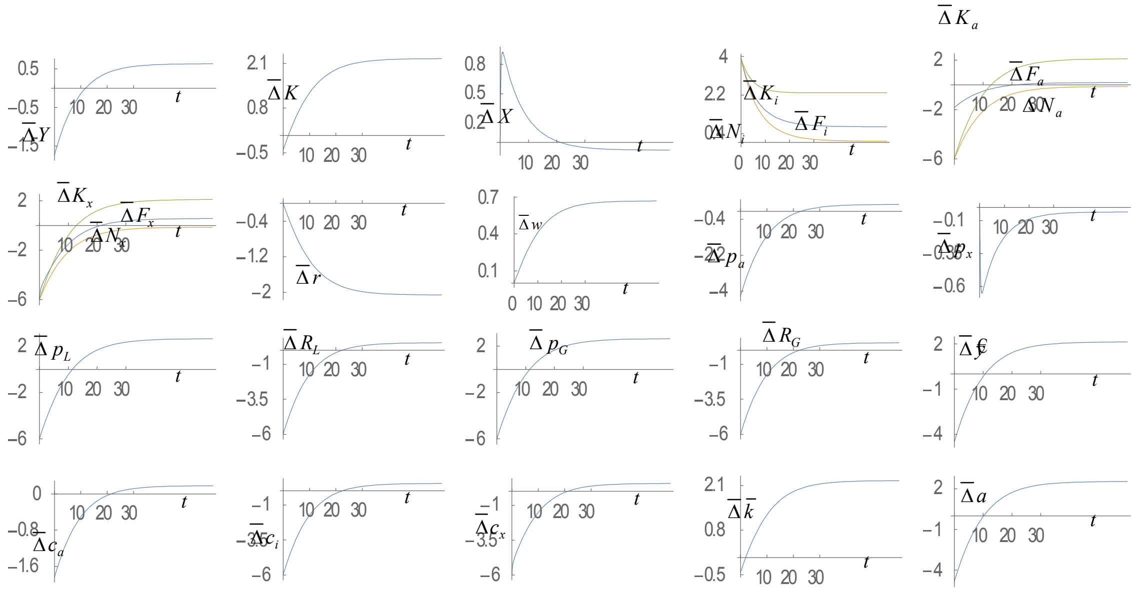

Fig. 7. The Propensity to Save Being Enhanced

The total factor productivity of the industrial sector being augmented

We now allow the total factor productivity of the industrial sector to change as follows: $A_i : 1 \Rightarrow 1.02$. The land use pattern is not affected. The effects on the other variables are plotted in Figure 8. The rate of interest rises initially and is not affected in the long term. The wage rate is enhanced. The national output rises. The national capital falls initially and rises in the long term. The resource stock rises initially and falls in the long term. The price of resource is enhanced. In the long term the prices and rents of gold and land are increased. The consumption levels of all the goods and wealth levels are enhanced in the long term.

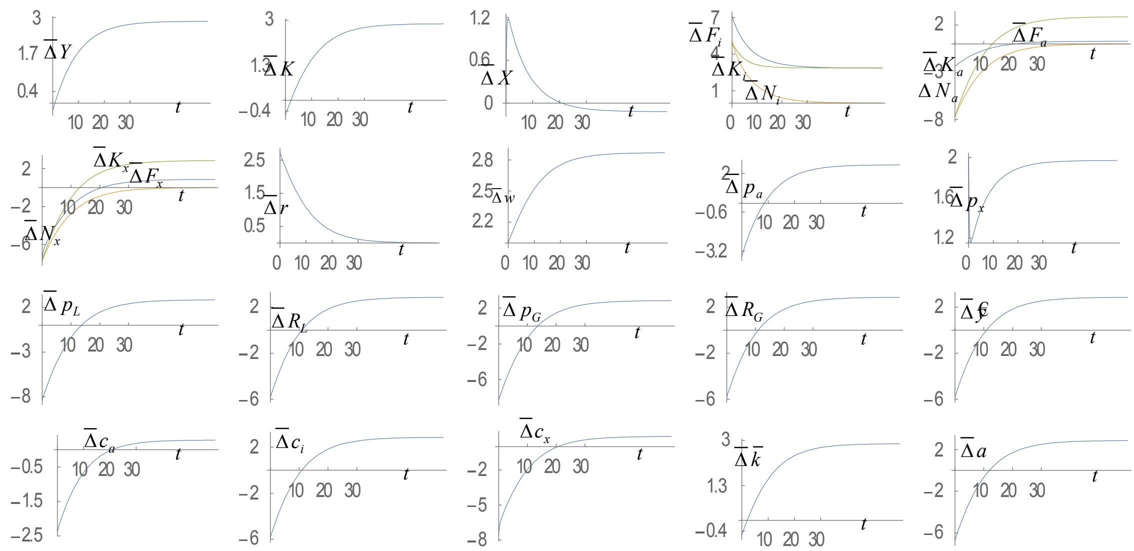

Fig. 8. The Total Factor Productivity of the Industrial Sector Being Augmented

5. Concluding remarks

This paper was concerned with a dynamic interdependence between values of gold, capital, land and renewable resource in a three sector growth model with endogenous wealth and renewable resources. We built the model by synthesizing the neoclassical growth theory, Ricardian theory and growth theory with renewable resources. The economic system consists of the industrial, agricultural, and resource sectors with given land and gold. The economic system is perfectly competitive. The model includes portfolio equilibrium between gold, land and physical wealth. We provided a computational procedure for simulating the model. The simulated case has a unique stable equilibrium point.

We plotted the motion of the dynamic system. We also conducted comparative dynamic analysis with regard to changes in the propensity to use gold, the propensity to consume resources, the propensity to consume housing, the propensity to consume agricultural goods, the propensity to consume industrial goods, the propensity to hold wealth, and the total factor productivity of the industrial sector population. Our model is built under many strict conditions without taking account of many possible important determinants of gold, land and resource prices.

These limitations become apparent in the light of the sophistication of the literature of growth theory, resource and land economics. We may extend or generalize the model. For instance, we may generalize the model by using more general function forms of the three sectors and the utility function. It is also possible to extend the model by taking account of heterogeneity of households. We may also relax the assumption of fixed land and the assumption of fixed amount of gold (Barro, 1979; Barsky and Summers, 1988; Davis and Heathcote, 2007). Gold mining is an important industry and new supply brings about changes in gold markets. It is possible to introduce endogenous gold supply into our 24odeling.

Appendix: Proving the Lemma

We now show that the dynamics can be expressed by two differential equations. From (2), (4) and (7), we get

$$z \equiv \frac{r + \delta_k}{w} = \frac{\tilde{\alpha}_i N_i}{K_i} = \frac{\tilde{\alpha}_a N_a}{K_a} = \frac{\tilde{\alpha}_x N_x}{K_x}, \tag{A1}$$

where $\tilde{\alpha}_j \equiv \alpha_j / \beta_j$, $j = i, a, x$. Equations (1) and (2) imply

$$r + \delta_k = \frac{\alpha_i A_i z^{\beta_i}}{\tilde{\alpha}_i^{\beta_i}}, \quad w = \frac{\tilde{\alpha}_i^{\alpha_i} \beta_i A_i}{z^{\alpha_i}}, \tag{A2}$$

where we also use (A1). We express w and r as functions of z. Equations (13) and (16) imply

$$\mu \text{₣} N = p_a F_a . \quad \text{(A3)}$$

From (4), we get

$$r + \delta_k = \frac{\alpha_a p_a F_a}{K_a} . \quad \text{(A4)}$$

Equations (A4) and (3) imply

$$p_a \left(\frac{L_a}{K_a} \right)^{\varsigma} = \frac{\tilde{\alpha}_a^{\beta_a} \left(r + \delta_k \right)}{\alpha_a A_a z^{\beta_a}} , \quad \text{(A5)}$$

where we use (A1). Equations (A3) and (A4) imply

$$\mu \text{₣} N = \left(\frac{r + \delta_k}{\bar{\tau}_a \alpha_a} \right) K_a . \quad \text{(A6)}$$

From (4) and (A3) we solve

$$R_L = \frac{\varsigma \mu \text{₣} N}{L_a} . \quad \text{(A7)}$$

Equations $R l_h = \eta \text{₣}$ in (13) and (A7) imply

$$\varsigma \mu N l_h = \eta L_a . \quad \text{(A8)}$$

Insert (24) in (23)

$$l_h N + (1 + \varphi) L_a = L . \quad \text{(A9)}$$

Equations (A8), (A9) and (24) imply

$$l_h = \frac{\eta L}{\eta N + (1 + \varphi) \varsigma \mu N} , \quad L_a = \frac{\varsigma \mu N l_h}{\eta} , \quad L_x = \varphi L_a . \quad \text{(A10)}$$

From (A10) we solve the land distribution. The land distribution is invariant over time.

The definition of ₣ implies

$$\text{₣} = (1 + r) \bar{k} + w + R_L \bar{l} + p_L \bar{l} + R_G \bar{g} + p_G \bar{g} . \quad \text{(A11)}$$

Insert (5) in (A11)

$$\tilde{y} = (1 + r)\bar{k} + w + \left(1 + \frac{1}{r}\right)\bar{l}\, R_L + \left(1 + \frac{1}{r}\right)\bar{g}\, R_G . \qquad \text{(A12)}$$

From (13) we have

$$R_L = \frac{\eta\, \tilde{y}}{l_h}, \quad R_G = \frac{\gamma\, \tilde{y}}{\tilde{g}}. \qquad \text{(A13)}$$

Insert (A13) in (A12)

$$\tilde{y} = \tilde{\omega}_1 \bar{k} + \tilde{\omega}_2 , \qquad \text{(A14)}$$

where

$$\tilde{\omega}_1 \equiv \tilde{\omega}_0 (1 + r), \quad \tilde{\omega}_2 \equiv \tilde{\omega}_0\, w, \quad \tilde{\omega}_0 \equiv \left[1 - \left(1 + \frac{1}{r}\right)\frac{\eta\, \bar{l}}{l_h} - \left(1 + \frac{1}{r}\right)\gamma\right]^{-1} .$$

From $R_L\, l_h = \eta\, \tilde{y}$ in (13) and (A14) we solve

$$R_L = \omega_1 \bar{k} + \omega_2 , \qquad \text{(A15)}$$

where

$$\omega_1 \equiv \frac{\eta\, \tilde{\omega}_1}{l_h}, \quad \omega_2 \equiv \frac{\eta\, \tilde{\omega}_2}{l_h}.$$

Insert (A1) in $N_i + N_a + N_x = N$

$$\frac{K_i}{\tilde{\alpha}_i} + \frac{K_a}{\tilde{\alpha}_a} + \frac{K_x}{\tilde{\alpha}_x} = \frac{N}{z}. \qquad \text{(A16)}$$

From (20) and (21) we have

$$K_i + K_a + K_x = N \bar{k}. \qquad \text{(A17)}$$

Insert (4) in (A3)

$$\mu\, \tilde{y} N = \left(\frac{r + \delta_k}{\alpha_a}\right) K_a . \qquad \text{(A18)}$$

From (A18) and (A14) we solve

$$K_a = \hat{\omega}_1 \bar{k} + \hat{\omega}_2, \tag{A19}$$

where

$$\hat{\omega}_1(z) \equiv \tilde{\omega}_1 \mu N \left(\frac{\alpha_a}{r + \delta_k} \right), \quad \hat{\omega}_2(z) \equiv \tilde{\omega}_2 \mu N \left(\frac{\alpha_a}{r + \delta_k} \right).$$

Insert (A19) in, respectively, (A16) and (A17)

$$\frac{K_i}{\tilde{\alpha}_i} + \frac{K_x}{\tilde{\alpha}_x} = b_1 \equiv \frac{N}{z} - \frac{\hat{\omega}_2}{\tilde{\alpha}_a} - \frac{\hat{\omega}_1 \bar{k}}{\tilde{\alpha}_a},$$
$$K_i + K_x = b_2 \equiv N \bar{k} - \hat{\omega}_1 \bar{k} - \hat{\omega}_2. \tag{A20}$$

Solve (A20)

$$K_i = \alpha_0 b_1 - \frac{\alpha_0 b_2}{\tilde{\alpha}_x}, \quad K_x = \frac{\alpha_0 b_2}{\tilde{\alpha}_i} - \alpha_0 b_1, \tag{A21}$$

where

$$\alpha_0 \equiv \left(\frac{1}{\tilde{\alpha}_i} - \frac{1}{\tilde{\alpha}_x} \right)^{-1}.$$

Insert the definitions of b_j in (A21)

$$K_i = m_i \bar{k} - \bar{m}_i, \quad K_x = m_x \bar{k} - \bar{m}_x, \tag{A22}$$

where

$$m_i(z) \equiv - \frac{\alpha_0 \hat{\omega}_1}{\tilde{\alpha}_a} - \frac{\alpha_0}{\tilde{\alpha}_x} N + \frac{\alpha_0}{\tilde{\alpha}_x} \hat{\omega}_1, \quad \bar{m}_i(z) \equiv \frac{\alpha_0 \hat{\omega}_2}{\tilde{\alpha}_a} - \frac{\alpha_0 N}{z} - \frac{\alpha_0}{\tilde{\alpha}_x} \hat{\omega}_2,$$
$$m_x(z) \equiv \frac{\alpha_0 N}{\tilde{\alpha}_i} - \frac{\alpha_0 \hat{\omega}_1}{\tilde{\alpha}_i} + \frac{\alpha_0 \hat{\omega}_1}{\tilde{\alpha}_a}, \quad \bar{m}_x(z) \equiv \frac{\alpha_0 \hat{\omega}_2}{\tilde{\alpha}_i} + \frac{\alpha_0 N}{z} - \frac{\alpha_0 \hat{\omega}_2}{\tilde{\alpha}_a}.$$

By (A19) and (A22), we solve the capital distribution as functions of z and $\bar{k}$. By (A1), we solve the labor distribution as functions of z and $\bar{k}$ as follows

$$N_i = \frac{z K_i}{\tilde{\alpha}_i}, \quad N_a = \frac{z K_a}{\tilde{\alpha}_a}, \quad N_x = \frac{z K_x}{\tilde{\alpha}_x}. \tag{A23}$$

From (13) and (8)

$$R_G = \frac{\gamma \text{₣}}{\text{₲}}, \quad p_G = \frac{R_G}{r}, \quad p_L = \frac{R_L}{r}. \qquad \text{(A24)}$$

Insert (13) in (17)

$$\chi \text{₣} N = \left(\frac{r + \delta_k}{\alpha_x}\right) K_x . \qquad \text{(A25)}$$

Insert (A14) and (A22) in (A25)

$$\bar{k} = \left(\tilde{\omega}_2 + \left(\frac{r + \delta_k}{\alpha_x \chi N}\right)\overline{m}_x\right)\left(\left(\frac{r + \delta_k}{\alpha_x \chi N}\right) m_x - \tilde{\omega}_1\right)^{-1} . \qquad \text{(A26)}$$

From (6) and (A24) we have

$$a = \varphi(z) \equiv \bar{k} + p_L \bar{l} + p_G \bar{g} . \qquad \text{(A27)}$$

It is straightforward to check that all the variables can be expressed as functions of z and X at any point in time as follows: r and w by (A2) $\rightarrow$ $\bar{k}$ by (A26) $\rightarrow$ K_a by (A19) $\rightarrow$ K_i and K_x by (A22) $\rightarrow$ N_i, N_x, and N_a by (A1) $\rightarrow$ ₣ by (A14) $\rightarrow$ $\bar{l}$ by (18) $\rightarrow$ $\bar{g} = \text{₲}$ by (19) and (20) $\rightarrow$ R_L by (A15) $\rightarrow$ p_L, R_G, and p_G by (A24) $\rightarrow$ a by (A27) $\rightarrow$ L_a, L_x and l_h by (A10) $\rightarrow$ p_a by (A5) $\rightarrow$ F_i by (1) $\rightarrow$ F_a by (3) $\rightarrow$ F_x by (6) $\rightarrow$ p_x by (7) $\rightarrow$ c_i, c_a, c_x, and s by (13). From this procedure, (5) and (14), we have

$$\dot{a} = \Lambda_0(z) \equiv s - a, \qquad \text{(A28)}$$

$$\dot{X} = \Omega(z, X) \equiv \phi_0 X\left(1 - \frac{X}{\phi}\right) - F_x . \qquad \text{(A29)}$$

Taking derivatives of (A27) with respect to t yields

$$\dot{a} = \frac{d\varphi}{dz}\dot{z}. \qquad \text{(A30)}$$

Equal (A28) and (A30)

$$\dot{z} = \Lambda(z) \equiv \Lambda_0\left(\frac{d\varphi}{dz}\right)^{-1} . \qquad \text{(A31)}$$

From (A29) and (A31), we determine the motion of z and X. We thus proved the lemma.

"The author declares that they have no conflict of interest".

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COMPETENCY MODELS AND THEIR USE IN PRACTICE

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Abstract. *The purpose of this article is to present the role of competency models in human resources management and to attempt an assessment of the number of organisations in Poland that have implemented such models, and of their effectiveness. The nature of the competency models' role has been assessed based on a literature review. The scale of use of competency models by organisations has been evaluated using desk research as the research technique.*

Sources of information consisted of the Study of Human Capital (Bilans Kapitału Ludzkiego) database by the Polish Agency for Enterprise Development (PARP) and reports from research conducted by the Institute of Business Development (IBR) and PARP. The conclusions are as follows: more