UDC 631.312 DOI 10.31548/energiya5(69).2023.109

STUDY OF THE MOVEMENT OF A MATERIAL PARTICLE ON A FLAT DISK ROTATING AROUND A PERPENDICULAR AXIS INCLINED TO THE HORIZON

S. F. Pylypaka, doctor of technical sciences, professor

A. V. Nesvidomin, Candidate of Technical Sciences, docent

National University of Life and Environmental Sciences of Ukraine

E-mail: a.nesvidomin@gmail.com

Abstract. *The movement of material particles along rotational planes is complex, since it should be considered as the result of the movement of the plane itself and the particle along this plane. The task becomes more difficult if the moving plane is inclined at a certain angle to the horizon. Its solution makes it possible to find out the regularities of the movement of a particle along an inclined plane, which rotates around an axis perpendicular to it.*

The purpose of the study is to establish the patterns of movement of material particles on a flat disc with and without blades, which rotates around a perpendicular axis inclined to the horizon.

If a round disk rotating around an axis perpendicular to it is located horizontally, then the kinematic parameters of the particle's motion on it do not depend on the point of impact of the particle on the disk. If the disk is tilted at a certain angle β to the horizon, it is obvious that the absolute trajectories of the particle's movement and other parameters of the movement will not be the same and will depend on the sector of the disk from which the particle starts its movement.

The relative and absolute motion of a particle on an inclined disk with and without rectilinear blades is considered. A system of differential equations of particle motion has been compiled using the accompanying trihedron of the transfer trajectory, which is a circle, and Frenet's formulas. Numerical integration of the system was carried out. The obtained results were visualized.

It was established that when particles hit an inclined disk that rotates around its own axis, the absolute trajectories of motion differ significantly from the trajectories of motion along a horizontal disk, and the difference in trajectories increases with an increase in the angle of inclination β. If rectilinear vanes are installed on the disc in the radial direction, the difference between the particle motion parameters will increase insignificantly as the angle β increases. When increasing the angular speed of rotation of the disk at a given angle, the shape of the absolute trajectories of particle motion practically does not change, but they are different depending on the point of impact on the disk. There is a certain area of impact and a certain sector of trajectories, after passing which the particle flies up after leaving the disc. Among this set, it is possible to analytically find the point of

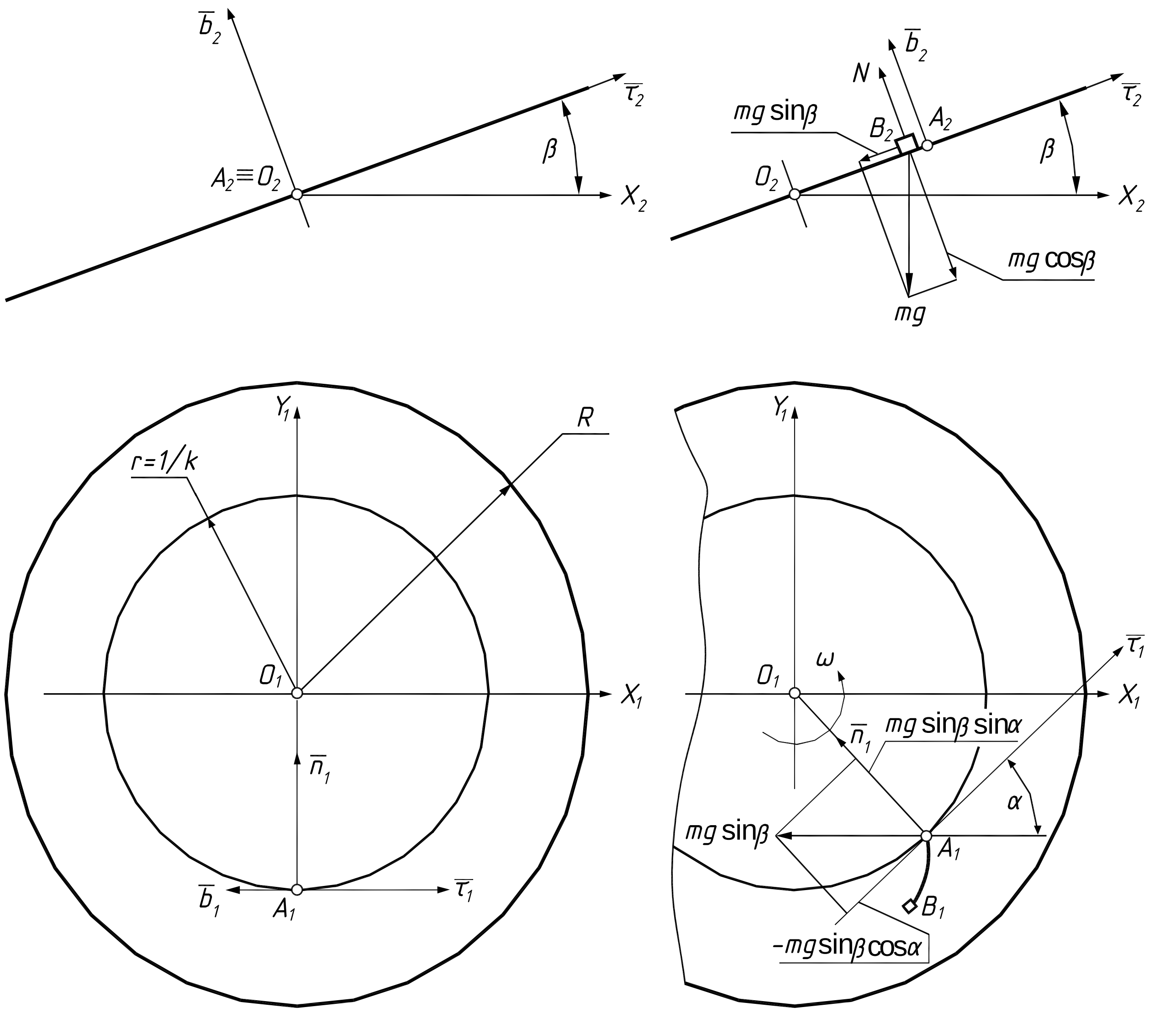
impact and the corresponding trajectory, which provide the maximum angle of elevation of the particle (equal to the angle β).
Key words: ***material particle, inclined disk, angle of inclination, angular velocity, trajectory of particle movement***

Topicality. The movement of material particles along rotational planes is complex, since it should be considered as the result of the movement of the plane itself and the particle along this plane. The task becomes more complicated if the moving plane is inclined at a certain angle to the horizon. Its solution makes it possible to find out the patterns of movement of a particle along an inclined plane, which rotates around an axis perpendicular to it.

Analysis of recent research and publications. The movement of a material particle on the rotating surfaces of the working bodies of agricultural machines is considered in the works of academicians of the Ukrainian Academy of Sciences P.M. Vasylenko [1] and P.M. Zayki [2]. In these works, the trajectories and other parameters of the movement of a particle that falls on a horizontal disk rotating around a vertical axis are investigated. The study of the general case of dispersion of mineral fertilizers by a centrifugal dispersing body was carried out in the work [3]. Dispersion of particles occurs more effectively when they fly upwards at a certain angle to the surface of the field when leaving the working body [4]. In part, this effect can be achieved by installing a flat rotating disc inclined to the horizontal plane.

The purpose of the study is to study the patterns of movement of material particles on a flat disk with and without blades, which rotates around a perpendicular axis inclined to the horizon.

Materials and methods of research. If a circular disk rotating around an axis perpendicular to it is located horizontally, then the kinematic parameters of the movement of a particle along it do not depend on the point of impact of the particle on the disk (it is not the distance from the axis of rotation, but the angle from a certain initial position that is meant). If the disk is tilted at a certain angle β to the horizon, it is obvious that the absolute trajectories of the particle's movement and other parameters of the movement will not be the same and will depend on the sector of the disk from which the particle starts its movement. In fig. 1, and the disk is located so that its plane is inclined at an angle β to the horizontal plane. Axes *Ox* and *Oy* lie in the plane of the disk, intersect in its center and are located so that the axis *Ox* and all lines parallel to it are the lines of greatest inclination of the disk, and the axis *Oy* is parallel to the horizontal plane. We will consider this system as stationary.



a *b*

Fig. 1. A flat disk assigned to two rectangular systems (stationary system *Oxy* and the moving system of the accompanying trihedron of a circle of radius $r = 1/k$):
a – mutual arrangement of systems before the start of movement;
b – decomposition of the forces acting on the particle in the system of the accompanying trihedron

Let's introduce another system - a moving accompanying trihedron of the trajectory of the transfer movement (circles of radius $r = 1/k$). It will rotate with the disk like a body rigidly attached to the disk. At the beginning of the movement, the unit orts $\bar{\tau}$ are $\bar{n}$ respectively parallel to the axes *Ox* and *Oy*, and the third ort $\bar{b}$ of the binormal is perpendicular to the disk plane (Fig. 1a).

Let's assume that before the start of the movement of the disk, the material particle is at point *A* - the origin of the coordinates of the accompanying trihedron. When the disk rotates, point *A* will move along a circle of radius *r*, and the particle will shift under the action of centrifugal force and take position *B* in the system of the accompanying trihedron (Fig. 1, b). If the position of the particle in the system of the accompanying trihedron is denoted by coordinates ρ_τ (projection of the distance to the particle on the ortho tangent $\bar{\tau}$) and ρ_n ρ_τ (projection of the distance to the particle on the orthos of the main normal $\bar{n}$), then the absolute acceleration in the projections on these orthos will be written [5]:

$$\begin{aligned} w_\tau &= v_A^2\left(\rho_\tau'' - k^2\rho_\tau - 2k\rho_n'\right); \\ w_n &= v_A^2\left(\rho_n'' - k^2\rho_n + 2k\rho_\tau' + k\right), \end{aligned} \tag{1}$$

where v_A is the speed of movement of the top of the trihedron in a circle of radius *r*. It can be written in terms of the angular speed of rotation of the disc: $v_A = \omega r$ or $v_A = \omega / k$, where *k* is the curvature of a circle of radius *r* (transfer trajectory). Differentiation in formulas (1) is carried out according to the arc coordinate *s* - the length of the arc of the transfer trajectory.

The weight of the particle at any point of the disk can be divided into two components: the component $mg \sin\beta$, which is in the plane of the disk and is directed along the line of greatest inclination, and the component $mg \cos\beta$, which acts on the plane of the disk in a perpendicular direction. Let's write down the differential equations of motion of the particle in the projections onto the orthos of the accompanying trihedron after it has turned to the angle α with respect to the fixed coordinate system:

$$\begin{aligned} mw_\tau &= -mg \sin\beta \cos\alpha - F_\tau; \\ mw_n &= mg \sin\beta \sin\alpha - F_n, \end{aligned} \tag{2}$$

where F_τ and F_n are the components of the friction forces in the projections on the corresponding vertices of the trihedron.

To find the friction force, we first find the reaction of the disk plane by projecting the acting forces onto the binormal $\bar{b}$:

$$N = mg\cos\beta. \tag{3}$$

If the coefficient of friction *f* of the particle on the disk is known, then the force of friction can be found from the expression:

$$F = fN = fmg\cos\beta. \tag{4}$$

Since the force of friction is directed tangentially to the trajectory of the relative motion, its components will be written on the $\bar{\tau}$ orths $\bar{n}$:

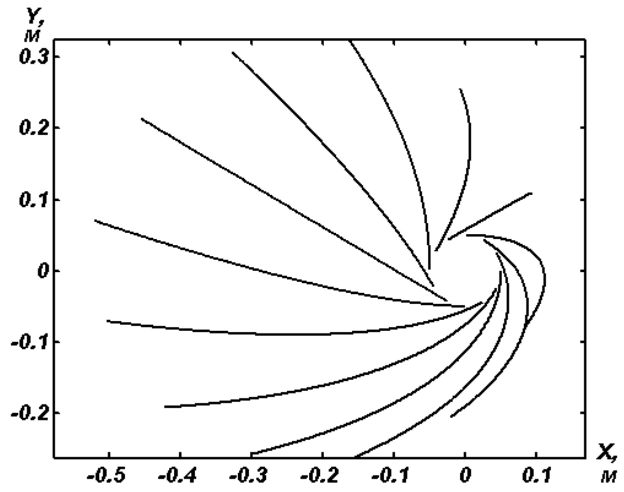
$$F_\tau = f\,\frac{mg\rho'_\tau}{\sqrt{\rho_\tau'^2 + \rho_n'^2}}; \qquad F_n = f\,\frac{mg\rho'_n}{\sqrt{\rho_\tau'^2 + \rho_n'^2}}. \tag{5}$$

By substituting the expressions of the components of the friction force (5) and absolute acceleration (1) in (2), and bearing in mind that $\alpha=ks$ and $v_A=\omega/k$, we obtain a system of second-order differential equations with two unknown functions: $\rho_\tau = \rho_\tau(s)$ and $\rho_n = \rho_n(s)$. After reduction by the mass *m* of the particle, they will be written:

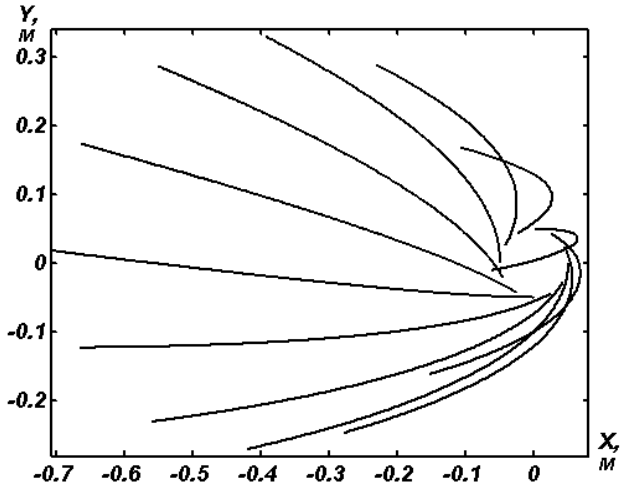
$$\begin{aligned} \rho''_\tau &= k^2\rho_\tau + 2k\rho'_n - \frac{gk^2}{\omega^2}\left(f\,\frac{\rho'_\tau\cos\beta}{\sqrt{\rho_\tau'^2+\rho_n'^2}} - \sin\beta\cos ks \right); \\ \rho''_n &= k^2\rho_n - 2k\rho'_\tau - k - \frac{gk^2}{\omega^2}\left(f\,\frac{\rho'_n\cos\beta}{\sqrt{\rho_\tau'^2+\rho_n'^2}} + \sin\beta\sin ks \right). \end{aligned} \tag{6}$$

Research results and their discussion. System (5) was solved by numerical methods. Dependencies $\rho_\tau = \rho_\tau(s)$ and $\rho_n = \rho_n(s)$ describe the trajectory of relative motion in the system of the accompanying trihedron. In this case (disc without blades), this trajectory will be a spiral. A more interesting task is the study of the absolute motion trajectory, which is described by the equations [6]:

$$\begin{aligned} x_B &= \rho_\tau\cos ks - \rho_n\sin ks + \frac{1}{k}\sin ks; \\ y_B &= \rho_\tau\sin ks + \rho_n\cos ks - \frac{1}{k}\cos ks. \end{aligned} \tag{7}$$



a



b

Fig. 2. Absolute trajectories of particle motion on an inclined disc without blades at $\omega=20\ s^{-1}$; $k=20\ m^{-1}$; $f=0.3$:

a – $\beta=30^{0}$; *b* – $\beta=60^{0}$

In fig. 2, the absolute trajectories of the particle motion along the disc are constructed according to equations (7) for $\beta=30^{0}$ and $\beta=60^{0}$ and under equal other conditions. The particle was fed to the disk at a distance of $r=0.05\ m$ from the center of rotation with an initial absolute velocity equal to the transfer velocity ($v_A=\omega r$) through 30^{0} along the disk's rotation. The trajectories are built for one complete revolution of the disk. From fig. 2, it can be concluded that as the angle *β increases*, the amount of particle lift decreases, approaching over time to the line of greatest inclination of the disk plane. If at the moment of delivery the absolute speed of the particle is zero, then it does not rise up at all. In fig. 3, the trajectories of the absolute movement of a particle with different coefficient of friction when it is fed to different points through a 90^{0} rotation of the disk are plotted. At $f=0$, in all cases the absolute trajectory is a straight line (marked by dots), which coincides with the line of greatest inclination. With an increase in the coefficient of friction, there is an increase in the deviation of the absolute trajectory from the line of greatest inclination.

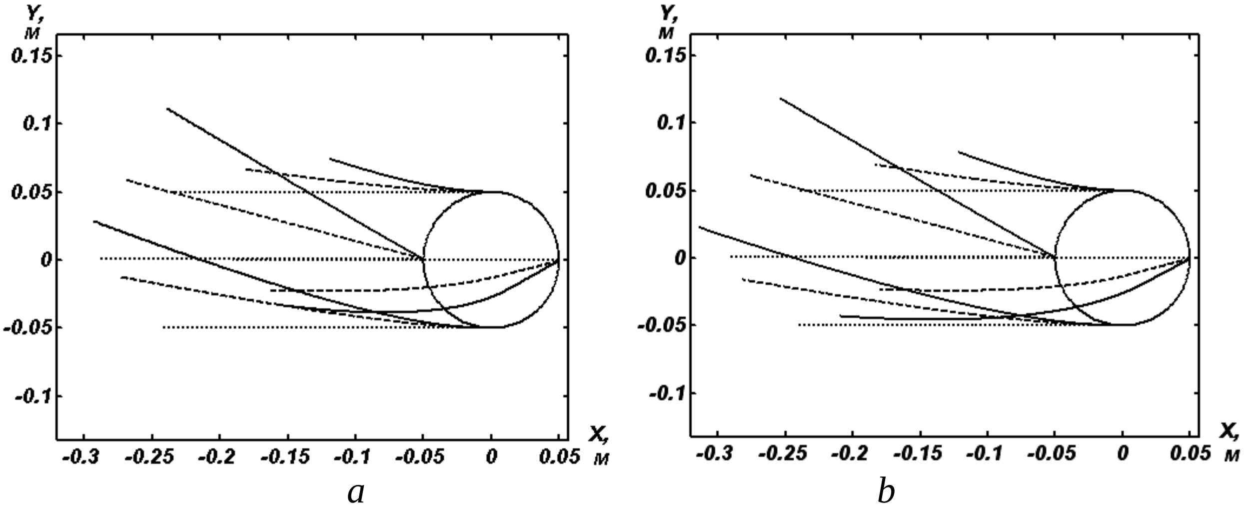


Fig. 3. Absolute particle motion trajectories at *f =0* (straight line, depicted by dots), *f =0.15* (dashed line), *f =0.3* (solid line) and) *β=30 ^{0}* :
a – ω=20 s^{-1}, the disc makes half a revolution;
b – ω=40 s^{-1}, the disc makes a complete revolution

It is interesting that with an increase in the angular speed of rotation of the disk, the shape of the absolute trajectory practically does not change, but the particle travels a shorter path in absolute motion. Trajectories in fig. 3,a and 3,b are almost the same, although in the first case the disk makes a half revolution, and in the second - a full revolution, but at twice the angular speed of rotation.

It is obvious that spreading process materials using an inclined disc without blades is impractical. Therefore, let's investigate the absolute particle trajectories for a disk with rectilinear blades fixed perpendicular to its plane in the radial direction. In this case, the relative movement is carried out along the center of the main normal $\bar{n}$. So, $\rho_\tau = \rho'_\tau = \rho''_\tau = 0$. The projections of the absolute acceleration of the particle according to (1) will be written:

$$w_\tau = -2v_A^2 k\rho'_n ; \qquad w_n = v_A^2\left(\rho''_n - k^2\rho_n + k\right). \tag{8}$$

Let's formulate the differential equation of motion of the particle. In the orthographic projection, $\bar{\tau}$ we have:

$$-2mv_A^2 k\rho'_n = -mg\sin\beta\cos ks + N_л , \tag{9}$$

from which the pressure force of the blade on the particle will be:

$$N_л = m\left(g\sin\beta\cos ks - 2v_A^2 k\rho'_n\right). \tag{10}$$

We will assume that the coefficient of friction of the particle on the disk and on the blade is the same. Then the differential equation of motion in the projection on the ort $\bar{n}$ will be written:

$$mv_A^2\left(\rho_n'' - k^2\rho_n + k\right) = mg\sin\beta\sin ks - fmg\cos\beta - fN_л . \tag{11}$$

After substituting (10) into (11), reducing by mass *m* and simplifying, we get:

$$\rho_n'' - 2fk\rho_n' - k^2\rho_n = \frac{gk^2}{\omega^2}\sin\beta\sin ks - \frac{fgk^2}{\omega^2}\left(\cos\beta + \sin\beta\cos ks\right) - k . \tag{12}$$

Equation (12) has an analytical solution:

$$\rho_n = \frac{1}{k} + f\,\frac{g\cos\beta}{\omega^2} + c_1 e^{\left(f-\sqrt{1+f^2}\right)ks} + c_2 e^{\left(f+\sqrt{1+f^2}\right)ks} + \\ + \frac{g\sin\beta}{2\omega^2\left(1+f^2\right)}\left[2f\cos ks - \left(1-f^2\right)\sin ks\right]. \tag{13}$$

The angle α of the rotation of the trihedron is determined through the arc coordinate *s* ($\alpha = ks$) or through time ($\alpha = \omega t$). Thus, by substituting the expression ω *t instead of ks* in equation (13), we proceed to the relative movement of the particle along the vane as a function of time: $\rho_n = \rho_n(t)$. Differentiating the obtained result by time *t*, we find the expression of the relative speed. Let's write down both expressions:

$$\rho_n(t) = \frac{1}{k} + f\,\frac{g\cos\beta}{\omega^2} + c_1 e^{\left(f-\sqrt{1+f^2}\right)\omega t} + c_2 e^{\left(f+\sqrt{1+f^2}\right)\omega t} + \\ + \frac{g\sin\beta}{2\omega^2\left(1+f^2\right)}\left[2f\cos\omega t - \left(1-f^2\right)\sin\omega t\right]; \tag{14}$$

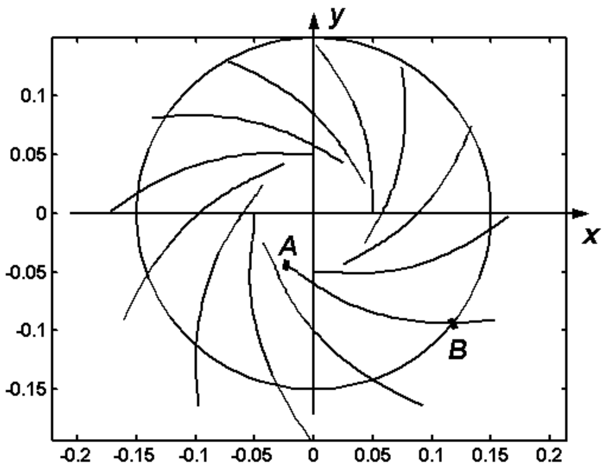
$$v = \frac{\rho_n(t)}{dt} = c_1\omega\left(f - \sqrt{1+f^2}\right)e^{\left(f-\sqrt{1+f^2}\right)\omega t} + c_2\omega\left(f + \sqrt{1+f^2}\right)e^{\left(f+\sqrt{1+f^2}\right)\omega t} - \\ - \frac{g\sin\beta}{2\omega\left(1+f^2\right)}\left[2f\sin\omega t + \left(1-f^2\right)\cos\omega t\right]. \tag{15}$$

Constant integrations c_1 and c_2 can be found under the condition that when a particle hits the disk, when the angle of its rotation from the zero position is the angle φ (it is equal to the angle α of the rotation of the trihedron), the distance of the relative movement $\rho_n = 0$ and the relative speed $v = 0$. If the disk is horizontal, then the rotation angle φ is irrelevant, since the results will be the same for any value. As a rule, it is accepted at $t = 0$, that is, $\varphi=\omega t$ will also be zero. For our case, its value is important, therefore $\varphi\neq 0$ and the expressions of integration constants in this case have a somewhat cumbersome form:

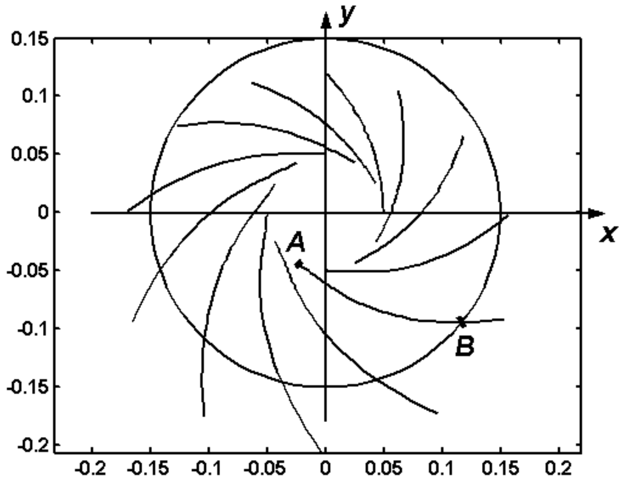
$$c_1=\frac{-e^{\left(-f+\sqrt{1+f^2}\right)\varphi}}{4k\omega^2\left(1+f^2\right)}\left\{\begin{aligned}&2\sqrt{1+f^2}\left(\sqrt{1+f^2}+f\right)\left(\omega^2+fgk\cos\beta\right)+\\&+gk\sin\beta\left[\left(f^2-1\right)\sin\varphi+2f\cos\varphi+\sqrt{1+f^2}\left(\cos\varphi+f\sin\varphi\right)\right]\end{aligned}\right\};$$

$$c_2=\frac{-e^{-\left(f+\sqrt{1+f^2}\right)\varphi}}{4k\omega^2\left(1+f^2\right)}\left\{\begin{aligned}&2\sqrt{1+f^2}\left(\sqrt{1+f^2}-f\right)\left(\omega^2+fgk\cos\beta\right)+\\&+gk\sin\beta\left[\left(f^2-1\right)\sin\varphi+2f\cos\varphi-\sqrt{1+f^2}\left(\cos\varphi+f\sin\varphi\right)\right]\end{aligned}\right\}. \quad (16)$$

Using formula (14) taking into account (16), the law of relative movement of a particle along the blade at different points of its impact on the disc (at different values of the angle φ*)* can be found. By substituting it into equation (7), we obtain the trajectory of the absolute motion of the particle (at the same time, in the indicated equations, you also need to switch to the time parameter *t*, that is, instead of *ks,* put *ωt,* and also keep in mind that $\rho_\tau=0$. Fig. 4 shows the absolute trajectories of the particle's motion along the surface of a disk with a diameter of *0.3 m* (*R =0.15 m*) when it hits the disk at a distance *r =1/ k* through 30^0. The direction of the *x axis* shows the rise of the plane of the disk, the *y axis* is parallel to the horizontal plane. The trajectories are constructed for the fourth It can be seen from Fig. 4 that in both cases ($\beta=30^0$, Fig. 4a and $\beta=60^0$, Fig. 4b) the largest angle of elevation of the particle when it leaves the disk will be in the case when it hits it at $\varphi=330^0$ (point *A*). At the moment of leaving the disk (point *B*), the trajectory is parallel to the *x axis*, that is, to the line of greatest inclination of the plane of the disk. Therefore, the angle of elevation of this particle is the largest among other particles and is equal to the angle *β*. From Fig. 4, a, b, it is also clear that the value of the angle *β* does not significantly affect the absolute trajectories of the particles. In particular, in both cases, the particle must hit the disc at $\varphi=330^0$ in order to reach an ascent angle equal to *β* when leaving it, while the disc rotates by 82.6^0.



a



b

Fig. 4. Absolute trajectories of the movement of particles falling on an inclined disk at a distance of *r=0.05 m* from the axis of rotation through *30^0* when it completes a quarter of a revolution. Output data: *ω=20 s^{-1}; k=20 m^{-1}; f=0.3*:
a – angle of inclination of the disk to the horizontal plane $\beta=30^0$;
b – angle of inclination of the disk to the horizontal plane $\beta=60^0$

According to formula (15), the speed of the relative movement of the particle along the blade can be found. In fig. 5, a graph of the relative movement of the particle from the moment it hits the disk (point *A*, Fig. 4) to the moment it leaves the disk (point *B*) is plotted. It can be seen from the graph that the change in the angle β of the inclination of the disk from $30^{\,0}$ to $60^{\,0}$ practically does not affect the relative speed, only when the disk is located horizontally ($\beta=0^0$) the speed changes slightly.

The absolute velocity of the particle upon exiting the disk is decisive, since the range of further flight depends on its value. It can be found in two ways: by differentiating over time the expression of the absolute trajectory (7) (under the condition that it is written as a function of time), or find it as a vector sum from its projections onto the vertices of the trihedron [6]:

$$\overline{v_B} = v_A\left[\bar{\tau}\left(1 - k\rho_n + \rho'_\tau\right) + \bar{n}\left(k\rho_\tau + \rho'_n\right)\right]. \quad (17)$$

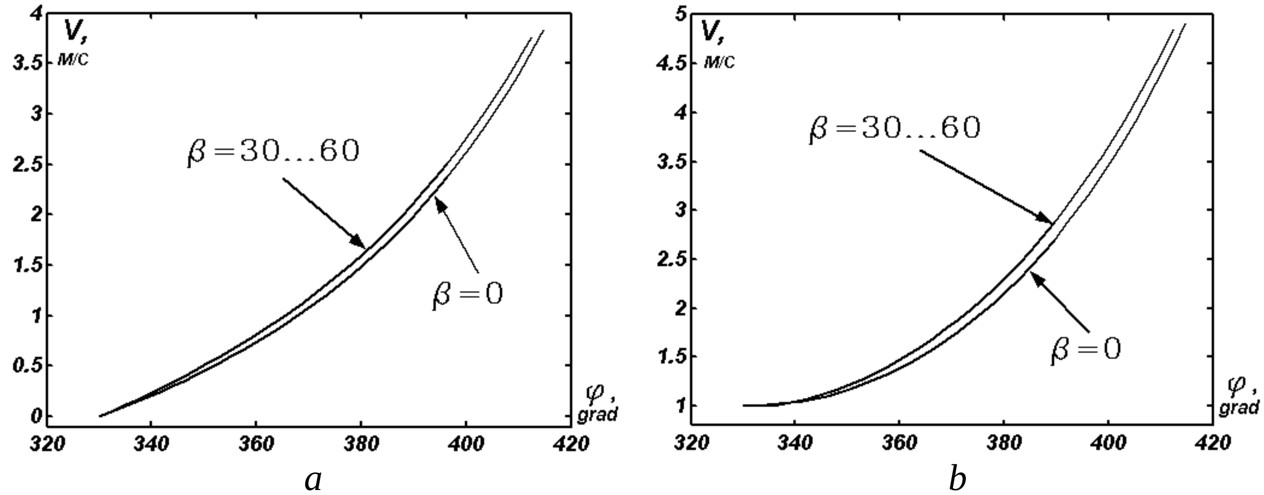


Fig. 5. Graphs of relative:
a – absolute *b* – velocities of a particle moving along a straight blade of an inclined disk ($\beta=0^{0}\ldots60^{0}$; $\omega=20\ s^{-1}$; $k=20\ m^{-1}$; $f=0.3$)

Formula (17) is general; for our case $\rho_\tau=\rho'_\tau=0$. In fig. 5, *b* graphs of absolute particle velocities as a function of the angle of φ rotation of the disk are plotted, the relative velocities of which are shown in fig. 5, a.

Let's increase the angular speed of rotation of the disk by two times and find the absolute trajectory and speed of movement of the particle. The corresponding graphs are shown in fig. 6.

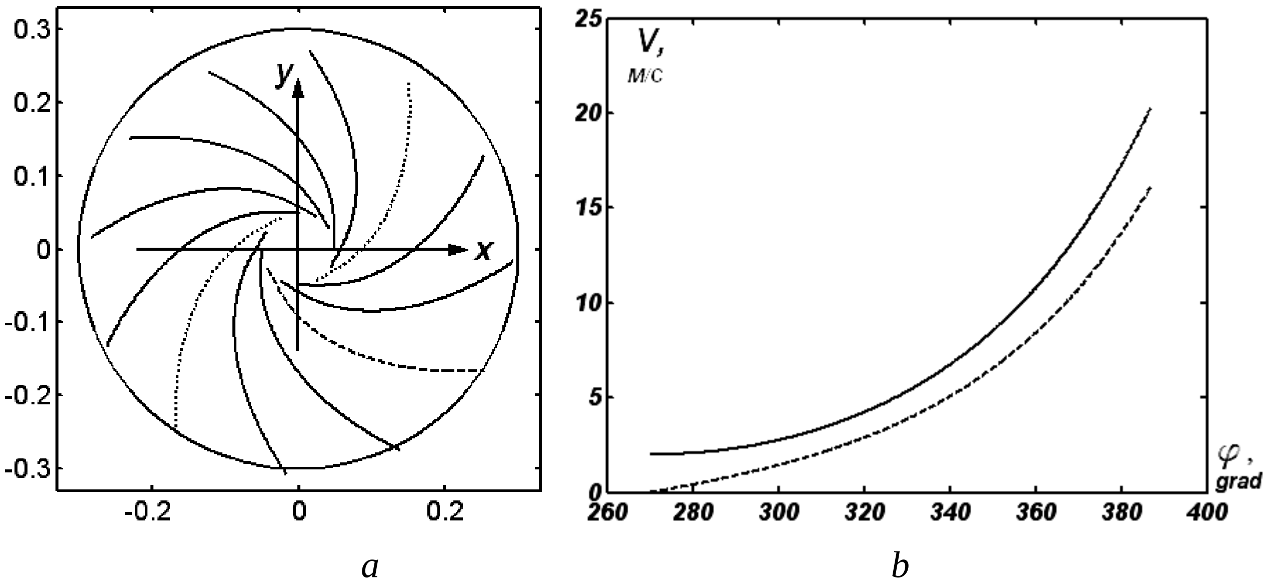


Fig. 6. Graphs of the dependence of the movement of a particle on an inclined disk at $\boldsymbol{\beta=45^{0}}$**;** $\boldsymbol{\omega=40\ s^{-1}}$**;** $\boldsymbol{k=20\ m^{-1}}$**;** $\boldsymbol{f=0.3}$**;** $\boldsymbol{R=0.3\ m}$**:**
a – graphs of absolute trajectories;
b – graphs of absolute (solid line) and relative velocities

The angular speed of rotation of the disc, but also other parameters were changed: the angle *β*, the radius of the disc *R*. All absolute trajectories (Fig. 6, a) are also constructed after 30^{0} when the disk is rotated by 117^{0}. In order for the angle of ascent of the particle when leaving the disk to be maximum, the particle must hit it at $\varphi=300^{0}$. The corresponding trajectory is shown by a dashed line. If the point of impact of the particle differs slightly from $\varphi=300^{0}$, then it will also fly upwards when leaving the disk, but at a smaller angle. In general, this sector of particle impact forms an angle from 30^{0} to -150^{0} (in Fig. 6, the extreme trajectories limiting it are marked with dotted lines). The tangents to the extreme trajectories at the point of exit from the disk are approximately parallel to the *y axis*, so the particles at the moment of exit from it will fly parallel to the horizontal plane. In fig. 6b shows the graphs of the relative and absolute velocities of one of the particles that hits the disk at $\varphi=270^{0}$, depending on the angle of its rotation.

Conclusions and perspectives. When particles hit an inclined disk rotating around its own axis, the absolute trajectories of motion differ significantly from the trajectories of motion along a horizontal disk, and the difference in trajectories increases with the increase of the inclination angle *β*. If rectilinear vanes are installed on the disk in the radial direction, then the difference between the parameters of the particle motion (absolute trajectory, relative and absolute velocities) increases slightly as the angle *β increases*. When increasing the angular speed of rotation of the disk at a given angle, *the shape of* the absolute trajectories of particle movement practically does not change, however, they are different depending on the point of impact on the disk (in the angular dimension). There is a certain area of impact and a certain sector of trajectories, after passing which the particle flies up after leaving the disc. Among this set, it is possible to analytically find the point of impact and the corresponding trajectory, which provide the maximum angle of elevation of the particle (equal to the angle *β*).

List of references

1. Василенко П. М. Теория движения частицы по шероховатым поверхностям сельскохозяйственных машин. К.: УАСХН, 1960. 283 с.

2. Заика П. М. Избранные задачи земледельческой механики. К.: Изд-во УСХА, 1992. 507 с.

3. Адамчук В. В. Дослідження загального випадку розгону мінеральних добрив відцентровим розсіювальним органом. Вісник аграрної науки. 2003. № 12. С. 51-57.

4. Адамчук В. В., Мойсеєнко В. К. Дослідження залежності дальності польоту мінеральних добрив від кута нахилу лопаток розсіювального органу до горизонтальної площини. Міжвідомчий тематичний наук. збірник „Механізація та електрифікація сільського господарства". Глеваха: ННЦ „ІМЕСГ", 2004. Вип. 88. С. 23 – 36.

5. Пилипака С. Ф. Теорія складного руху матеріальної точки на площині. Частина друга. Абсолютне прискорення. Задачі на динаміку точки. Електротехніка і механіка. 2006. №2. С. 88 - 100.

6. Пилипака С. Ф. Теорія складного руху матеріальної точки на площині. Частина перша. Абсолютна швидкість і траєкторія. Електротехніка і механіка. 2006. №1. С. 84 - 94.

References

1. Vasylenko, P. M. (1960). Teoriya dvizheniya chastitsy po sherokhovatym poverkhnostyam sel′skokhozyaystvennykh mashin [Theory of movement of particles on rough surfaces of agricultural machines]. Kyiv: UASKHN, 283.

2. Zaika, P. M. (1992). Izbrannyye zadachi zemledel′cheskoy mekhaniki [Selected tasks of agricultural mechanics]. Kyiv: Publishing House of the Ukrainian Academy of Sciences, 507.

3. Adamchuk, V.V. (2003). Doslidzhennia zahalnoho vypadku rozghonu mineralnykh dobryv vidtsentrovym rozsiiuvalnym orhanom [Investigation of the general case of dispersal of mineral fertilizers by a centrifugal dispersing body]. Herald of Agrarian Science, 12, 51-57.

4. Adamchuk, V. V., Moiseyenko, V. K. (2004). Doslidzhennia zalezhnosti dalnosti polotu mineralnykh dobryv vid kuta nakhylu lopatok rozsiiuvalnoho orhanu do horyzontalnoi ploshchyny [Study of the dependence of the flight range of mineral fertilizers on the angle of inclination of the blades of the dispersing body to the horizontal plane]. Interdepartmental Thematic Science. collection "Mechanization and Electrification of Agriculture". Glevakha: NNC "IMESG", 88, 23-36.

5. Pylypaka, S. F. (2006). Doslidzhennia zalezhnosti dalnosti polotu mineralnykh dobryv vid kuta nakhylu lopatok rozsiiuvalnoho orhanu do horyzontalnoi ploshchyny. [The theory of complex motion of a material point on a plane. Part two. Absolute acceleration. Problems on point dynamics]. Electrical engineering and mechanics, 2, 88 - 100.

6. Pylypaka, S. F. (2006). Teoriia skladnoho rukhu materialnoi tochky na ploshchyni. Chastyna persha. Absoliutna shvydkist i traiektoriia. {The theory of complex motion of a material point on a plane. Part one. Absolute speed and trajectory]. Electrical engineering and mechanics, 1, 84 - 94.

ДОСЛІДЖЕННЯ РУХУ МАТЕРІАЛЬНОЇ ЧАСТИНКИ ПО ПЛОСКОМУ ДИСКУ, ЯКИЙ ОБЕРТАЄТЬСЯ НАВКОЛО ПЕРПЕНДИКУЛЯРНОЇ ОСІ, НАХИЛЕНОЇ ДО ГОРИЗОНТУ

С. Ф. Пилипака, А. В. Несвідомін

Анотація. *Рух матеріальних частинок по ротаційних площинах є складним, оскільки його слід розглядати як результат руху самої площини і частинки по цій площині. Задача стає складнішою, якщо рухома площина нахилена під певним кутом до горизонту. Її розв'язання дає можливість з'ясувати закономірності руху частинки по похилій площині, яка обертається навколо перпендикулярної до неї осі.*

Мета дослідження – встановити закономірності руху матеріальних частинок по плоскому диску з лопатками і без, який обертається навколо перпендикулярної осі, нахиленої до горизонту.

Якщо круглий диск, що обертається навколо перпендикулярної до нього осі, розташований горизонтально, то кінематичні параметри руху частинки по ньому не залежать від точки попадання частинки на диск. Якщо ж диск нахилити під певним кутом β до горизонту, то очевидно, що абсолютні траєкторії руху частинки і інші параметри руху не будуть однакові і залежатимуть від сектора диска, із якого частинка розпочинає свій рух.

Розглянуто відносний та абсолютний рухи частинки по похилому диску з прямолінійними лопатками та без них. Складено систему диференціальних рівнянь руху частинки із застосуванням супровідного тригранника переносної траєкторії, якою є коло, та формул Френе. Здійснено чисельне інтегрування системи. Зроблено візуалізацію одержаних результатів.

Встановлено, що при попаданні частинок на похилий диск, який обертається навколо власної осі, абсолютні траєкторії руху значно відрізняються від траєкторій руху по горизонтальному диску, причому відмінність у траєкторіях зростає із збільшенням кута нахилу β. Якщо на диск встановити прямолінійні лопатки у радіальному напрямі, то різниця між параметрами руху частинки зросте несуттєво при збільшенні кута β. При збільшенні кутової швидкості обертання диска при заданому куті форма абсолютних траєкторій руху частинок практично не змінюється, однак вони є різними в залежності від точки попадання на диск. Існує певна область попадання і певний сектор траєкторій, після проходження яких частинка після сходу із диска летить вгору. Серед цієї множини можна знайти аналітичним способом точку попадання і відповідну траєкторію, які забезпечують максимальний кут підйому частинки (рівний куту β).

Ключові слова: ***матеріальна частинка, похилий диск, кут нахилу, кутова швидкість, траєкторія руху частинок***