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RESEARCH OF THE MOVEMENT OF A MATERIAL PARTICLE ON THE SURFACE OF AN OBLIQUE HELICOID UNDER THE ACTION OF THE FORCE OF OWN WEIGHT

S. F. Pylypaka, Doctor of Technical Sciences, Professor

A.V. Nesvidomin, Candidate of Technical Sciences, Docent

National University of Life and Environmental Sciences of Ukraine

E-mail: a.nesvidomin@gmail.com

Abstract. *The problem that describes the movement of a particle along a helical surface is used in the design of spiral separators. Modeling the motion of a material particle on helical surfaces and its investigation by modern methods of numerical integration and visualization makes it possible to obtain a real picture of motion in the absence of full-scale models of such surfaces. This makes it possible to search for helical surfaces to improve their operational characteristics.*

The purpose of the study was to establish the regularity of the movement of a material particle along an oblique helicoid depending on the structural parameters of the surface.

The oblique helicoid has two design parameters - the pitch h and the angle β of the inclination of the rectilinear generating surfaces to the horizontal plane. With the help of these parameters, there are more opportunities to influence the nature of the movement of a particle along an oblique helicoid compared to a helical conoid and a wide helicoid.

The differential equations of motion of a material particle on the surface of an oblique helicoid under the action of its own weight were formulated. The equations are solved by numerical methods. The obtained results were visualized

It has been established that the movement of a material particle with a known coefficient of friction along the surface of an oblique helicoid can be ensured at a given distance from its axis by combinations of structural parameters of the surface. At the same time, the resolution of the surface during the separation of particles with different friction coefficients practically does not change. However, in the transition period (before the stabilization of the motion), there is a significant difference in the trajectories of particle motion, which increases as the angle of inclination of the rectilinear generating surfaces decreases. This gives reason to consider material separation at the stage of the transition period, which requires further theoretical and experimental research.

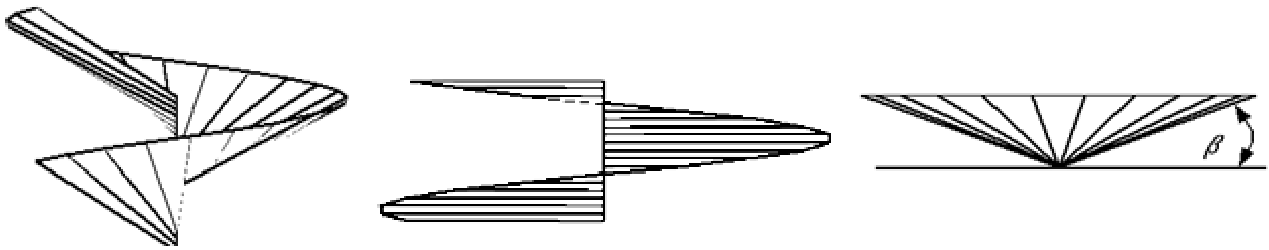
Key words: *oblique helicoid, material particle, trajectory of motion*

Introduction. The problem of the theory of the movement of a particle on a helical surface is currently solved in a sufficiently complete and generalized form [1] . Its solution was determined by the requests of practice in the design of spiral separators. Despite the fact that such separators are passive working bodies and do not require energy to drive them, they also have disadvantages. This is relatively low performance and low resolution (a small difference in the trajectories of particles with different physical and mechanical properties) [2] . Modeling the motion of a material particle on helical surfaces and its investigation by modern methods of numerical integration and visualization makes it possible to obtain a real picture of motion in the absence of full-scale models of such surfaces. This makes it possible to search for helical surfaces to improve their operational characteristics.

Analysis of recent research and publications. The movement of a material particle along an oblique (unfolding) helicoid is considered in the works of Prof. M.I. Akimova [1] and Prof. P. M. Zaiki [3]. M.I. Sysoeva gave a generalized solution to the problem of particle movement along a helical surface of constant pitch, the axial section of which is an arbitrary curve [4]. As a special case, he considered an expanding helicoid, as well as an oblique helicoid (a surface whose axial section is a straight line inclined at a certain angle to the horizontal plane). All the listed problems are solved in the cylindrical coordinate system. Academician P.M. Vasilenko indicated the possibility of solving similar problems in the system of the accompanying trihedron of the particle's trajectory (the so-called natural coordinate system). In paper [5], in the natural coordinate system, the problem of the movement of a material particle along an unexpanded helical surface with a horizontal arrangement of rectilinear generators (helical conoid or auger) was solved, and in paper [6] – along an expanded helicoid formed by a set of rectilinear generators tangent to the helical line

The purpose of the study – is to reveal the regularities of the movement of a material particle along an oblique helicoid depending on the structural parameters of the surface.

Materials and methods of research. There are two structural parameters of an oblique helicoid (Fig. 1, a) - step h and the angle β of inclination of the rectilinear generating surfaces to the horizontal plane. The previously mentioned surfaces (screw conoid and expanding helicoid) each have one design parameter. In a helical conoid (Fig. 1, b) $\beta=0$, so there remains one parameter - the step h , and in a spreading helicoid, the angle β is equal to the angle of elevation of the original helical line, that is, there is a certain dependence between h and β [6]. Thus, there are more opportunities to influence the nature of the movement of a particle along an oblique helicoid with the help of these parameters in comparison with the named helical surfaces. In general, a particle cannot move along a helical conoid for a long time. It first accelerates, moving away from the axis of the conoid, then slows down and finally stops [5]. This is explained by the fact that as it moves away from the axis of the conoid, the angle of inclination of the trajectory to the horizontal plane decreases and the moment comes when the particle can no longer overcome the force of friction. It is obvious that at small (close to zero) values of the angle, β such motion of the particle should also be characteristic of an oblique helicoid.



a *b* *c*

Fig. 1. Linear surfaces (only the frontal projection is shown), built according to equations (2):
a – oblique helicoid ($b \neq 0; \beta \neq 0$); *b* – helical conoid ($b \neq 0; \beta = 0$);
c – cone ($b = 0; \beta \neq 0$)

Results of the studies and their discussion. To find the trajectory of the movement of the particle along the surface and its speed v , we project the basic equation of the dynamics of the point $m\bar{a} = \bar{F}$,, where m is the mass of the point (particle), , $\bar{a}$ - the acceleration given to it by the force of acceleration $\bar{F} = mg$ of the particle (g =9.81 m/s ^{2}), on the axis of the accompanying of the Darboux trihedron of the trajectory. The Darboux trihedron is a moving coordinate system, the unit vertices of which are attached to the surface and located in space as follows. We take a point on the trajectory of the particle and draw a plane tangent to the surface through it. In this plane, one ortho $\bar{T}$ coincides with the direction of movement, that is, it is tangent to the trajectory, and the second $\bar{P}$ is perpendicular to $\bar{T}$. The third ort $\bar{N}$ perpendicular to the first two and is normal to the surface. The main equation of the dynamics of a point in the projections onto the orthos of the accompanying Darbou trihedron is reduced to a system of differential equations (the detailed derivation of these equations is shown in the work [6]):

$$\begin{cases} v\dfrac{dv}{ds} = g\cos\psi - f\,(g\cos\omega + v^2 k_н); \\ v^2 k_г = g\cos\varphi, \end{cases} \tag{1}$$

where f is the coefficient of friction; s is the length of the arc of the trajectory on the surface; ψ, φ,ω - the angles between the particle weight vector and each of the vertices of the trihedron $\overline{T}$, . $\overline{P}$, $\overline{N}$ the Darboux trihedron, respectively; k_n and k_d are the normal and geodesic curvature of the trajectory, respectively. m is not included in system (1), since the equations were reduced to it. System (1) describes the movement of a material particle on the surface in the general case, while the angles ψ, φ, ω, speed v, geodesic k_g and normal k_n curvature of the trajectory are functions of its arc s or another parameter that defines the curve on the surface.

Let the oblique helicoid be given by the parametric equations [3]:

$$X = \rho\cos\alpha; \qquad Y = \rho\sin\alpha; \qquad Z = b\alpha + \rho\cdot \mathrm{tg}\beta, \tag{2}$$

where $b = h/2\pi$ - helical parameter of the surface – a constant value; ρ and α- independent variable surfaces defining a point on an oblique helicoid - the distance from the axis of the helicoid to the point and the angle of rotation around the axis, respectively.

If the independent variables ρ are α connected by a certain dependence, then a line will be given on the surface (2). In order for this line to be the trajectory of the particle's motion, it is necessary to find such a relationship between ρ and α that the conditions of system (1) are fulfilled.

The normal curvature k_n of the trajectory included in system (1) is determined by the coefficients of the first and second quadratic forms, and the geodesic k_g – by the coefficients of only the first quadratic surface form. Let's find the coefficients of the first quadratic form of the surface (2). Partial derivatives and coefficients G , F , E will be:

$$\frac{\partial X}{\partial \alpha} = X_\alpha = -\rho \sin\alpha; \qquad \frac{\partial X}{\partial \rho} = X_\rho = \cos\alpha;$$
$$\frac{\partial Y}{\partial \alpha} = Y_\alpha = \rho\cos\alpha; \qquad \frac{\partial Y}{\partial \rho} = Y_\rho = \sin\alpha;$$
$$\frac{\partial Z}{\partial \alpha} = Z_\alpha = b; \qquad \frac{\partial Z}{\partial \rho} = Z_\rho = \mathrm{tg}\beta; \tag{3}$$
$$G = X_\alpha^2 + Y_\alpha^2 + Z_\alpha^2 = \rho^2 + b^2; \qquad F = X_\alpha X_\rho + Y_\alpha Y_\rho + Z_\alpha Z_\rho = b\,\mathrm{tg}\beta;$$
$$E = X_\rho^2 + Y_\rho^2 + Z_\rho^2 = \frac{1}{\cos^2\beta}.$$

The grid of coordinate lines of the surface (2) consists of two families: one family is rectilinear generating lines, the other is helical lines. This grid is not orthogonal because the mean *F coefficient* is not zero. For further work, let's simplify the first quadratic form, moving to an orthogonal grid. To do this, we will leave the family of rectilinear generators unchanged and find a family of orthogonal trajectories to it. In this case, it is much easier to find the expression of the geodesic curvature of the trajectory, and the second quadratic form of the surface is also simplified.

To find orthogonal trajectories to the family of rectilinear generators, it is necessary to solve the differential equation:

$$Ed\rho + Fd\alpha = 0. \tag{4}$$

By substituting the values of the corresponding coefficients from (3) into (4), after solving, we obtain:

$$\rho = u - b\alpha\sin\beta\cos\beta, \tag{5}$$

where *u* is the integration constant. By assigning a specific value to the constant *u* and substituting (5) into (2), we obtain a line perpendicular to the rectilinear generators of the oblique helicoid. Since there can be many such values and each of them will have its own line, we take *u* as a new independent variable instead of ρ. Thus, after substituting (5) into (2), we obtain the equation of an oblique helicoid, referred to an orthogonal grid of coordinate lines:

$$X = u\cos\alpha - b\alpha\sin\beta\cos\beta\cos\alpha;$$
$$Y = u\sin\alpha - b\alpha\sin\beta\cos\beta\sin\alpha; \tag{6}$$
$$Z = u\cdot\mathrm{tg}\beta + b\alpha\cos^2\beta.$$

Finally, so that the parameter u is not proportional to the length of the rectilinear generator, but equal to it, the directional vector of the generator must be normalized, that is, to go to the directional cosines. To do this, we find the module of the vector, the components of which are the expressions in equations (6) for the variable u. Having obtained the square root of the sum of their squares, we obtain: $1 / \cos\beta$. Dividing each component by the found expression, we obtain direction cosines and equation (6) takes the final form:

$$\begin{aligned} X &= \cos\beta(u - b\alpha\sin\beta)\cos\alpha; \\ Y &= \cos\beta(u - b\alpha\sin\beta)\sin\alpha; \\ Z &= b\alpha + \sin\beta(u - b\alpha\sin\beta). \end{aligned} \tag{7}$$

To find the length of a line on a surface and its geodesic and normal curvatures, it is necessary to have expressions of the first and second quadratic forms. Let's find the first, second and mixed partial derivatives of equations (7):

$$\begin{aligned} X_\alpha &= -\cos\beta[b\sin\beta\cos\alpha + (u - b\alpha\sin\beta)\sin\alpha]; \qquad & X_u &= \cos\beta\cos\alpha; \\ Y_\alpha &= -\cos\beta[b\sin\beta\sin\alpha - (u - b\alpha\sin\beta)\cos\alpha]; \qquad & Y_u &= \cos\beta\sin\alpha; \\ Z_\alpha &= b\cos^2\beta; \qquad & Z_u &= \sin\beta; \end{aligned}$$

$$\begin{aligned} X_{\alpha\alpha} &= \cos\beta[2b\sin\beta\sin\alpha - (u - b\alpha\sin\beta)\cos\alpha]; \qquad & X_{uu} &= 0; \\ Y_{\alpha\alpha} &= -\cos\beta[2b\sin\beta\cos\alpha + (u - b\alpha\sin\beta)\sin\alpha]; \qquad & Y_{uu} &= 0; \\ Z_{\alpha\alpha} &= 0; \qquad & Z_{uu} &= 0; \end{aligned} \tag{8}$$

$$X_{\alpha u} = -\cos\beta\sin\alpha; \qquad Y_{\alpha u} = \cos\beta\cos\alpha; \qquad Z_{\alpha u} = 0.$$

The coefficients G , F , E of the first quadratic form will be:

$$\begin{aligned} &G = X_\alpha^2 + Y_\alpha^2 + Z_\alpha^2 = \cos^2\beta[b^2 + (u - b\alpha\sin\beta)^2]; \\ &F = X_\alpha X_u + Y_\alpha Y_u + Z_\alpha Z_u = 0; \quad E = X_u^2 + Y_u^2 + Z_u^2 = 1. \end{aligned} \tag{9}$$

The coefficients N , L , M of the second quadratic form will be:

$$N=\frac{1}{\sqrt{GE-F^2}}\begin{vmatrix} X_{\alpha\alpha} & Y_{\alpha\alpha} & Z_{\alpha\alpha}\\ X_{\alpha} & Y_{\alpha} & Z_{\alpha}\\ X_{u} & Y_{u} & Z_{u}\end{vmatrix}=-\frac{\sin\beta\cos\beta\left[2b^2+(u-b\alpha\sin\beta)^2\right]}{\sqrt{b^2+(u-b\alpha\sin\beta)^2}};$$

$$M=\frac{1}{\sqrt{GE-F^2}}\begin{vmatrix} X_{\alpha u} & Y_{\alpha u} & Z_{\alpha u}\\ X_{\alpha} & Y_{\alpha} & Z_{\alpha}\\ X_{u} & Y_{u} & Z_{u}\end{vmatrix}=\frac{b\cos\beta}{\sqrt{b^2+(u-b\alpha\sin\beta)^2}}; \qquad (10)$$

$$L=\frac{1}{\sqrt{GE-F^2}}\begin{vmatrix} X_{uu} & Y_{uu} & Z_{uu}\\ X_{\alpha} & Y_{\alpha} & Z_{\alpha}\\ X_{u} & Y_{u} & Z_{u}\end{vmatrix}=0.$$

Let's find the first and second quadratic forms of the surface (7) and their ratio - the normal curvature:

$$I=ds^2=Edu^2+2Fdud\alpha+Gd\alpha^2=du^2+\cos^2\beta\left[b^2+(u-b\alpha\sin\beta)^2\right]d\alpha^2; \qquad (11)$$

$$II=Ldu^2+2Mdud\alpha+Nd\alpha^2=\cos\beta\frac{2bdud\alpha-\sin\beta\left[2b^2+(u-b\alpha\sin\beta)^2\right]d\alpha^2}{\sqrt{b^2+(u-b\alpha\sin\beta)^2}}; \qquad (12)$$

$$k_н=\frac{II}{I}=\frac{\left\{2bdud\alpha-\sin\beta\left[2b^2+(u-b\alpha\sin\beta)^2\right]d\alpha^2\right\}\cos\beta}{\left\{du^2+\cos^2\beta\left[b^2+(u-b\alpha\sin\beta)^2\right]d\alpha^2\right\}\sqrt{b^2+(u-b\alpha\sin\beta)^2}}. \qquad (13)$$

In order for the surface (7) to have a given line, it is necessary to establish a certain relationship between the variables u and α. We will consider the variable u as a function of the variable α: $u=u(\alpha)$. Then the first quadratic form (11), which is a linear element of the surface and the normal curvature k_n (13) will be written:

$$\left(\frac{ds}{d\alpha}\right)^2=u'^2+\cos^2\beta\left[b^2+(u-b\alpha\sin\beta)^2\right]; \qquad (14)$$

$$k_н=\frac{\left\{2bu'-\sin\beta\left[2b^2+(u-b\alpha\sin\beta)^2\right]\right\}\cos\beta}{\left\{u'^2+\cos^2\beta\left[b^2+(u-b\alpha\sin\beta)^2\right]\right\}\sqrt{b^2+(u-b\alpha\sin\beta)^2}}. \qquad (15)$$

where u' is the derivative of the variable u with respect to the parameter α. The geodesic curvature of a line on a surface refers to the internal properties of the surface and can be determined through the coefficients of the first quadratic form. For a surface with a coefficient $E=1$, the geodesic curvature can be found using the formula [7]:

$$k_г = \frac{\sqrt{G}}{\left(u'^2 + G\alpha'^2\right)^{3/2}}\left(u''\alpha' - \alpha''u' - \frac{1}{2}G_u\alpha'^3 - \frac{1}{2}\frac{G_\alpha}{G}u'\alpha'^2 - \frac{G_u}{G}u'^2\alpha'\right), \qquad (16)$$

where G_α, G_u - partial derivatives of the coefficient G according to the corresponding parameter:

$$G_u = 2\cos^2\beta(u - b\alpha\sin\beta); \qquad G_\alpha = -2b\cos^2\beta\sin\beta(u - b\alpha\sin\beta). \qquad (17)$$

Formula (16) implies that the line on the surface is given by the dependence between the variables u and α through the third parameter t : $u=u(t)$; $\alpha=\alpha(t)$. We will assume that the dependence between the parameters u and α is given directly in the form $u=u(\alpha)$, therefore $\alpha'=1$, $\alpha''=0$. Let's substitute these values, as well as expressions (17) and coefficient G from (9) into (16). We will get:

$$k_г = \cos\beta\times$$
$$\times\frac{\left[b^2 + (u - b\alpha\sin\beta)^2\right]\cdot\left[u'' - \cos^2\beta(u - b\alpha\sin\beta)\right] + u'(u - b\alpha\sin\beta)(b\sin\beta - 2u')}{\left\{u'^2 + \cos^2\beta\left[b^2 + (u - b\alpha\sin\beta)^2\right]\right\}^{3/2}\sqrt{b^2 + (u - b\alpha\sin\beta)^2}}. \qquad (18)$$

To substitute in system (1), we have to find expressions for the angles ψ, φ, ω, each of which is formed by the direction of the weight force mg and one of the orthos of the Darboux trihedron. Since the direction of gravity coincides with the *OZ axis* fixed coordinate system , then the specified angles will be defined as the angles between the direction of the *OZ axis* and the corresponding ortho: (for ψ- ort $\bar{T}$; for φ- ort $\bar{P}$; for ω-ort $\bar{N}$). The weight force vector { *0, 0, 1* } is directed downward parallel to the *OZ axis* , while we will assume that the *OZ axis* itself is also directed downward (this corresponds to the nature of the movement, since the αZ coordinate of the trajectory increases with the increase of the parameter, and at the same time the particle moves downward). In this case, the cosine of the corresponding angle will be the direction cosine of the corresponding orthogonal to the *OZ axis* , that is, the projection of this orthogonal onto the *OZ axis* .

Let's find an expression for the cosine of the angle ψ. Projections of the tangent vector to the trajectory on the axis of the fixed coordinate system are derivatives of equations (7) with respect to the variable α:

$$x' = \cos\beta\left[(u' - b\sin\beta)\cos\alpha - (u - b\alpha\sin\beta)\sin\alpha\right];$$
$$y' = \cos\beta\left[(u' - b\sin\beta)\sin\alpha + (u - b\alpha\sin\beta)\cos\alpha\right]; \quad (19)$$
$$z' = b + (u' - b\sin\beta)\sin\beta.$$

From the projections (19), we find the expression for the cosine of the angle ψ:

$$\cos\psi = \frac{z'}{\sqrt{x'^2 + y'^2 + z'^2}} = \frac{b + (u' - b\sin\beta)\sin\beta}{\sqrt{u'^2 + \cos^2\beta\left[b^2 + (u - b\alpha\sin\beta)^2\right]}}. \quad (20)$$

To determine the angle, ω we find the direction of the normal to the surface in projections on the axis of the fixed coordinate system $OXYZ$. Since the normal to the surface is perpendicular to the vectors tangent to the coordinate lines, it can be found from the vector product of these vectors:

$$\overline{N} = \begin{vmatrix} X & Y & Z \\ X_u & Y_u & Z_u \\ X_\alpha & Y_\alpha & Z_\alpha \end{vmatrix}, \qquad \text{звідки} \qquad \begin{aligned} N_x &= Y_u Z_\alpha - Y_\alpha Z_u; \\ N_y &= -X_u Z_\alpha + X_\alpha Z_u; \\ N_z &= X_u Y_\alpha - X_\alpha Y_u. \end{aligned} \quad (21)$$

Substituting the partial derivatives from (8) into (21), we obtain the coordinates of the vector directed along the normal to the surface:

$$N_x = \cos\beta\left[b\sin\alpha - \sin\beta(u - b\alpha\sin\beta)\cos\alpha\right];$$
$$N_y = \cos\beta\left[b\cos\alpha + \sin\beta(u - b\alpha\sin\beta)\sin\alpha\right]; \quad (22)$$
$$Z_z = (u - b\alpha\sin\beta)\cos^2\beta.$$

Using the coordinates (22), we find the expression for the cosine of the angle ω:

$$\cos\omega = \frac{N_z}{\sqrt{N_x^2 + N_y^2 + N_z^2}} = \frac{(u - b\alpha\sin\beta)\cos\beta}{\sqrt{b^2 + (u - b\alpha\sin\beta)^2}}. \quad (23)$$

Finally, to find out the expression for the cosine of the angle φ, it is necessary to know the coordinates of the vector $\overline{P}$. Since it is perpendicular to the two vectors $\overline{N}$ and $\overline{T}$, then its coordinates will be determined from the vector product of the specified vectors:

$$\overline{P} = \begin{vmatrix} X & Y & Z \\ x' & y' & z' \\ N_x & N_y & N_z \end{vmatrix}, \qquad \text{звідки} \qquad \begin{aligned} P_x &= y'N_z - z'N_y; \\ P_y &= -x'N_z + z'N_x; \\ P_z &= x'N_y - y'N_z. \end{aligned} \quad (24)$$

Substituting expressions (19) and (22) into (24), we obtain the coordinates of the vector directed along the centroid $\overline{P}$:

$$P_x = \cos\beta\{Au'\sin\alpha + [A^2\cos^2\beta + b(b\cos^2\beta + u'\sin\beta)]\cos\alpha\};$$
$$P_y = \cos\beta\{-Au'\cos\alpha + [A^2\cos^2\beta + b(b\cos^2\beta + u'\sin\beta)]\sin\alpha\}; \qquad (25)$$
$$P_z = -[bB - A^2\sin\beta]\cos^2\beta, \qquad \text{де} \qquad A = u - b\alpha\sin\beta,\ \ B = u' - b\sin\beta.$$

By projections (25), we find the expression for the cosine of the angle φ:

$$\cos\varphi = \frac{P_z}{\sqrt{P_x^2 + P_y^2 + P_z^2}} =$$
$$= -\frac{(bB - A^2\sin\beta)\cos\beta}{\sqrt{A^2(u'^2 + 2b^2\cos^2\beta) + A^4\cos^2\beta + b^2(b\cos^2\beta + u'\sin\beta)^2 + (bB\cos\beta)^2}}. \qquad (26)$$

Now we have all the expressions included in the system (1) in the function of one variable α. We substitute in (1) the expression *ds* from (14), the expressions for the curvatures k_n from (15) and k_g from (18), and the expressions for the cosines of the angles from (20), (23), and (26). After simplifications, let's write the system in a form convenient for integration in the *MatLab environment* using the *Simulink dynamic systems modeling package :*

$$\begin{cases} u'' = A\cos^2\beta + \dfrac{Au'(2u' - b\sin\beta)}{A^2 + b^2} - \qquad (27) \\ -\dfrac{g(bB - A^2\sin\beta)[u'^2 + (A^2 + b^2)\cos^2\beta]^{3/2}}{v^2\sqrt{A^2 + b^2}\sqrt{A^2(u'^2 + 2b^2\cos^2\beta) + A^4\cos^2\beta + b^2(b\cos^2\beta + u'\sin\beta)^2 + (bB\cos\beta)^2}}; \\ v' = \dfrac{g}{v}(b + B\sin\beta) - f\dfrac{\cos\beta}{\sqrt{A^2 + b^2}}\left[v\dfrac{2bu' - (2b^2 + A^2)\sin\beta}{\sqrt{u'^2 + (A^2 + b^2)\cos^2\beta}} + \dfrac{g}{v}A\sqrt{u'^2 + (A^2 + b^2)\cos^2\beta}\right], \end{cases}$$

where *A* and *B* are expressions of variables, the designations of which are given in (25). The system of differential equations (27) includes two unknown functions: $u = u(\alpha)$ and $v = v(\alpha)$. The integration of such a system by numerical methods is shown in [5] using the appropriate model. It includes three integrators, each of which must be set to constant integration. Three constant integrations set the initial conditions under which the movement of the particle begins, namely: u_o sets the position of the particle on the surface at the initial moment of movement (distance *u* along the rectilinear generator); v_o – initial speed; u_0'- the direction of movement at the initial moment, which is specified in the curvilinear coordinate system of the surface. We will consider the movement of particles with initial velocities close to zero (v_o cannot be set equal to zero, since inadmissible division will occur in system (27)), so the direction of entry of the particle onto the surface is of no significant importance. When the initial speed is significantly greater than zero, then its direction at the initial moment of movement at a given point on the surface plays a significant role, as it affects the further trajectory of movement. This is shown in the work [6] on the example of the movement of a particle along a unfolding helicoid.

If the initial velocity of the particle is zero or close to it, then its movement along the surface begins in the direction of the line of greatest inclination. For an unfolding helicoid and for other unfolding surfaces with the same inclination of the generators, this direction coincides with the rectilinear generator surface at all its points, and for non-expanded surfaces, to which the oblique helicoid belongs, the rectilinear generator is not the line of greatest inclination [8]. Therefore, the initial trajectory of a particle along an oblique helicoid will differ from a similar trajectory along an expanded helicoid. Integration of system (27) showed that at small values of the angle, β the movement of a particle along an oblique helicoid is similar to the movement of a particle along a helical conoid [5], i.e., after acceleration, it slows down its movement and stops. The question arises: at what minimum value of the angle β will the particle not stop? The answer to this question can be given based on the following considerations. When $b=0$, equation (2) describes the surface of the cone (Fig. 1, c). If the angle β of inclination of the elements of the cone to the horizontal plane (which in this case is the angle of greatest inclination) is smaller than the angle of friction of the particle, then its movement along the cone under the influence of its own weight will become impossible. True, at $b \neq 0$, that is, when the cone is transformed into the surface of an oblique helicoid, the angle of the greatest inclination increases, but as it moves away from the axis of the helicoid, it approaches the angle, that β is, to the limit, after which movement becomes impossible. This confirmed the integration of system (27) at an angle β smaller than the friction angle. In this case the movement of the particle is possible and stabilized provided that the particle hits the surface of the helicoid at a certain distance ρ_o from its axis. As the distance $\rho_{o\ decreases}$, the particle accelerates more and more and the moment comes when it moves along the helicoid away from its axis to such a distance that further movement becomes impossible. The particle will not stop even at small values $\rho_{of\ o}$ if the angle β is greater than the friction angle, i.e. β>arctg f . In this case, its motion stabilizes after a certain time [3] and it then moves along a helical line at a constant distance ρ from the axis with a constant speed v .

Let's find the dependencies v and ρ from the structural parameters of the surface β and b and the coefficient of friction f. The distance ρ can be found from the equations of the surface (7) according to the Pythagorean theorem:

$$\rho = \sqrt{X^2 + Y^2} = \cos\beta(u - b\alpha\sin\beta). \qquad (28)$$

Solving (28) with respect to u, we find the expression $u = u(\alpha)$ and its derivatives:

$$u = \frac{\rho}{\cos\beta} + b\alpha\sin\beta; \qquad u' = b\sin\beta; \qquad u'' = 0. \qquad (29)$$

Substituting expressions (29) and $v' = 0$ into system (27), after simplifications we obtain:

$$\begin{cases} v^2 = -g\rho\cdot \mathrm{tg}\beta; \\ b\cos\beta\sqrt{\rho^2 + b^2} = f\rho\sqrt{\rho^2 + b^2\cos^2\beta}. \end{cases} \qquad (30)$$

The first expression of the system (30) was obtained in work [4] without the "minus" sign (in our case, the *OZ axis* is directed downward, so the angle β should be taken negative). This expression allows you to find the speed of the particle after the stabilization of the motion, when it moves at a constant distance ρ from the axis. The same distance ρ can be found from the second equation of system (30):

$$\rho^2 = b^2\cos\beta\frac{(1-f^2)\cos\beta + \sqrt{(1-f^2)\cos^2\beta + 4f^2}}{2f^2}. \qquad (31)$$

In work [3] Acad. P.M. Zaika also gives the expression of the distance ρ depending on the structural parameters of the surface and the coefficient of friction (p. 335, formula (6.4.30), in which a slightly different notation of the structural parameters is adopted). It differs from (31), but it is possible to show their identity, which was done by the authors of this article.

If in (31) we consider the coefficient of friction f to be constant, then it can be seen that the distance ρ depends on two design parameters: the angle β and the value b . Therefore, the same distance value ρ can be obtained from (31) with different combinations of design parameters b and β. If these parameters are considered variables, then equation (31) will clearly describe the surface, the horizontal sections of which will be curves establishing the relationship between b and β for a certain value ρ. The construction of surfaces in the form of graphs with the drawing of horizontal sections (isolines) with the indication of the value of the function at the points of these isolines is provided by modern software products. In fig. 2, according to equation (31), such a surface is constructed when the structural parameters *b are changed* and β within the given limits using the *MatLab system*.

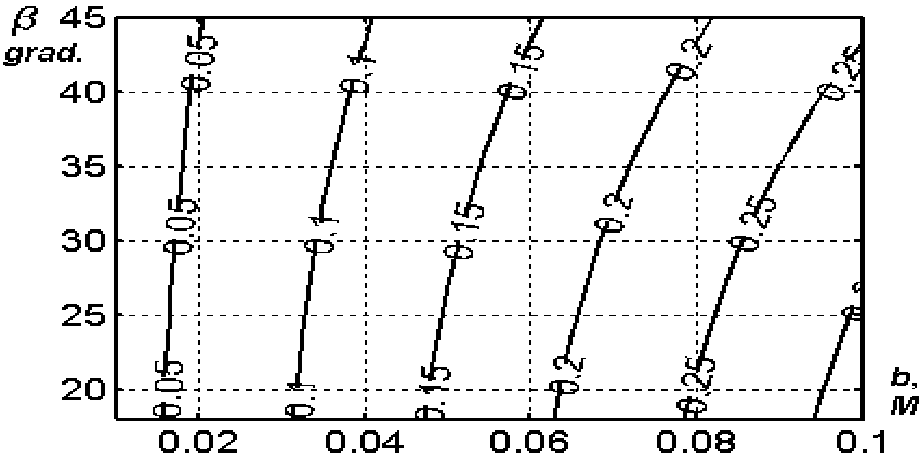


Fig. 2. Curved lines establishing the relationship between the design parameters b and β the oblique helicoid for a given distance ρ:
f = 0.3; b =0.01...0.1 m; $\beta=18^{0}...45^{0}$

As can be seen from Fig. 2, the given value $\rho=0.1$ *m* corresponds to a helicoid with different values of b and angle β. For the lower limit of the angle β taken $\beta=18^{0}$ (since arctg $0.3 = 16.7^{0}$ and at smaller values of the angle β the particle can stop) screw parameter b =0.0314 *m* (the exact value is found by equation (31)). For the upper limit $\beta=45^{0}$ corresponds to the value b =0.04095 *m*. In the first case, the step is $h = 2\pi b = 2 \cdot 3.14 \cdot 0.0314 = 0.1973$ *m* , in the second - $h = 2 \cdot 3.14 \cdot 0.04095 = 0.2576$ *m* . For other combinations of b and β, which provide $\rho=0.1$ *m* , the step will be within the specified limits, that is, within *20-26 cm* . If we are talking about the separation of agricultural materials, then the question of choosing the value arises ρ. Its value depends on the size of the separating surface (diameter and pitch of the helicoid), as well as the speed of movement of particles and their dispersion on the surface at different coefficients of friction. For the established motion, these parameters can be determined by formulas (30) (first expression) and (31). If there is a question about the moment of stabilization of motion, then to answer it you need to integrate the system of differential equations (27).

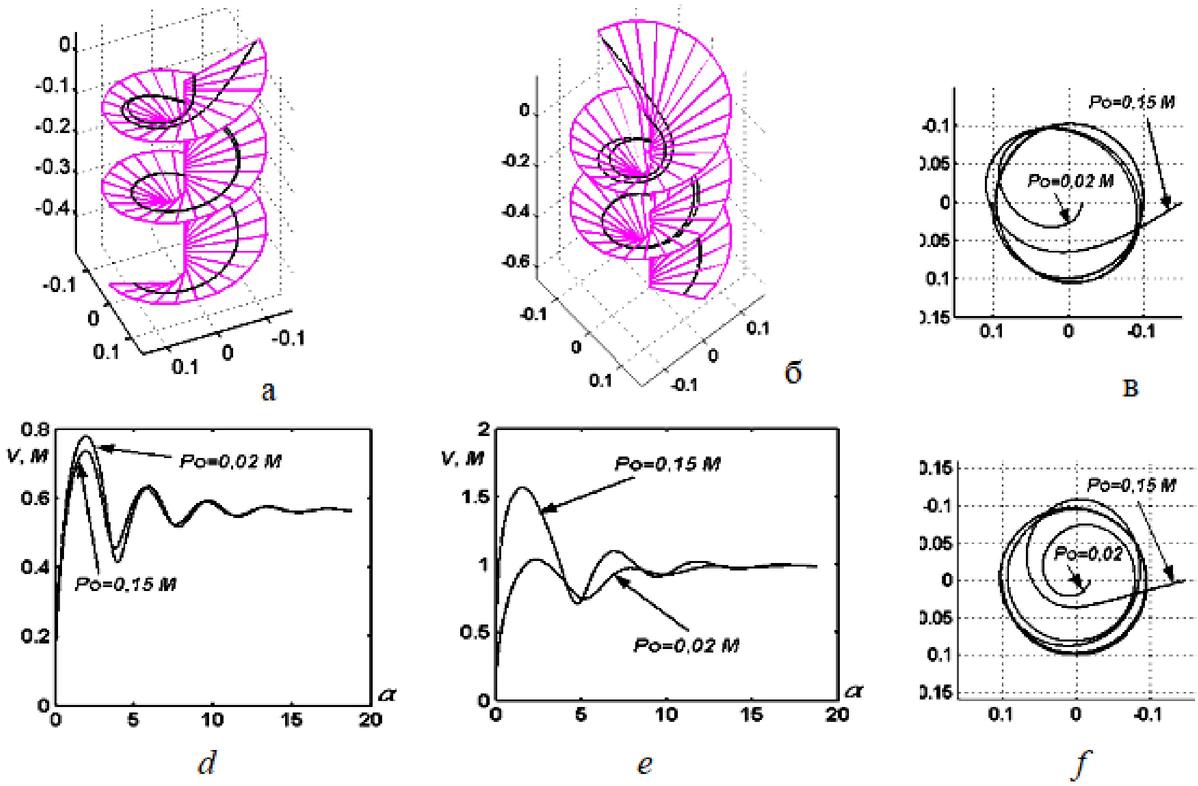


Fig. 3. Graphical illustrations for the integration of the system of differential equations (27) of the motion of a particle at f =0.3 and $v_o \approx 0$:

a trajectories of particle movement at ρ_o = *0.02 m* and ρ_o = *0.15 m* along an oblique helicoid with design parameters b = *0.0314* and β= *18* ^{0};

b trajectories of particle movement at ρ_o = *0.02 m* and ρ_o = *0.15 m* along an oblique helicoid with design parameters b = *0.04095* and β= *45* ^{0};

c horizontal projections of particle movement along an oblique helicoid with structural parameters b =*0.0314* and β=*18* ^{0};

d the graph of the change in the speed of particle movement $v = v(\alpha)$ with the structural parameters of the helicoid b = *0.0314* and β= *18* ^{0};

e the graph of the change in the speed of particle movement $v = v(\alpha)$ with the design parameters of the helicoid b =*0.04095* and β=*45* ^{0};

f horizontal projections of particle movement along an oblique helicoid with design parameters *b =0.04095* and β*=45* ^{0}

In fig. 3 shows the results of system integration (27) with the help of graphs, and Fig. 3a and 3b, the movement trajectory is superimposed on the surface image. Initial conditions were taken for ρ*=0.1 m* for the previously mentioned combinations of design parameters *b* and β. The movement of the particle started at different distances from the axis of the helicoid and stabilized after approximately two and a half turns in both cases, which corresponds to a rotation angle *of 15 rad* on the velocity graphs (Fig. 3d and 3e) $\alpha\approx$. It can be seen from the same graphs that the velocities are approaching constant values, the values of which can be more accurately calculated by the first expression of system (30).

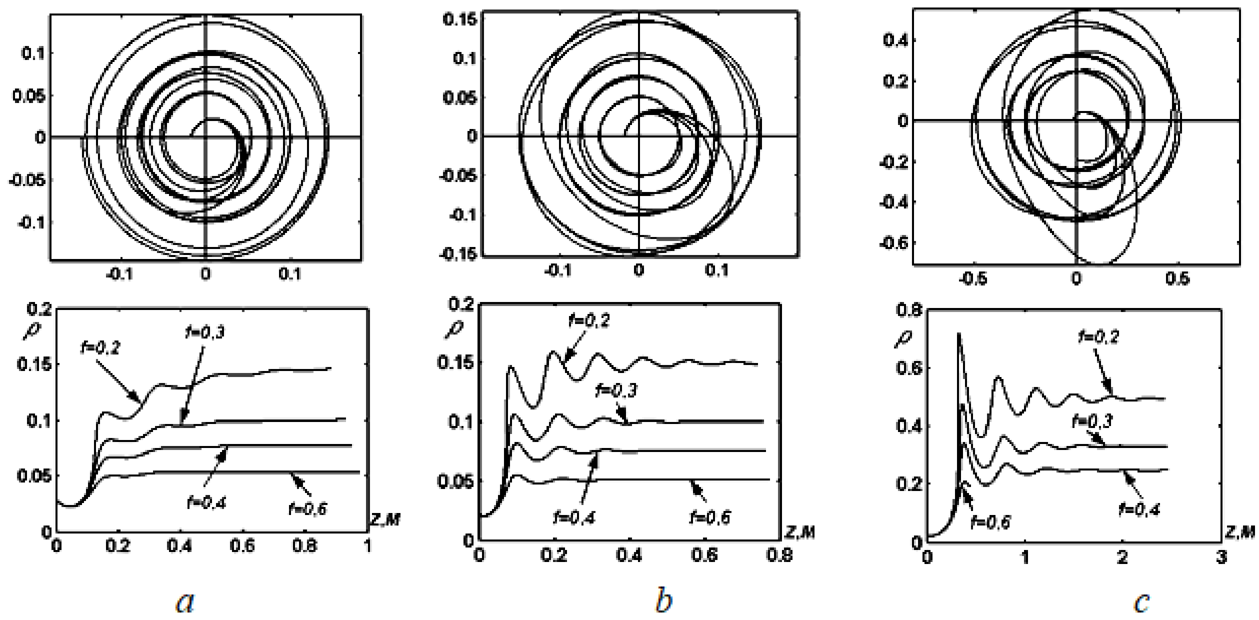


Fig. 4. Horizontal projections of particle motion trajectories with different friction coefficients (above) and their corresponding dependences $\rho = \rho(z)$ (below) for a helicoid with the following design parameters:
a - β\u003d 45 ^{0}*; b = 0.04095 m;* *b - β\u003d 18* ^{0}*; b =0.0 314 m;* c - *β\u003d 10* ^{0}*; b =0, 1 m*

Let 's find out how the particles are distributed on the surface of the oblique helicoid if the friction coefficients are different. In fig. 4 above shows horizontal projections of the movement of particles with different coefficients of friction, and below – graphs of distance dependencies ρ from the height of descent *z*. In all cases, the movement of particles begins at a distance of ρ=0.02 m. In fig. 4,a and 4,b show the previously considered cases at β=45 ^{0} and β=18 ^{0} respectively, and in fig. 4, in – at β=10 ^{0}. Analyzing figures 4a and 4b, it can be concluded that after stabilization of motion, particles with different friction coefficients come out at approximately the same distance from the axis of the helicoid. However, as the angle decreases, β their movement in the transition period differs in that the amplitude of the fluctuations of the distance ρ from the axis of the helicoid increases. At β=10 ^{0} (Fig. 4,c), the particle with the friction coefficient f =0.6 accelerated, and then stopped at a distance of $\rho\approx$0.2 m , which was mentioned earlier.

The analysis of the figures above shows that at small angles, β it is during the transition period (before the stabilization of motion) that the trajectories of particles with different friction coefficients differ significantly from each other, i.e., in this period, the resolution of the surface is quite large. Therefore, it is legitimate to consider the issue of material separation at the stage of the transition period, but this requires further theoretical research and experimental tests.

In conclusion, we note that conducting such research has become possible in modern conditions, when existing software products allow not only to integrate systems of differential equations by numerical methods, but also to perform the second no less important stage - to visualize the obtained results.

Conclusions. Theoretical studies have shown that the movement of a material particle with a known coefficient of friction on the surface of an oblique heclicoid can be ensured at a given distance from its axis by combinations of structural parameters of the surface. At the same time, the resolution of the surface during the separation of particles with different friction coefficients practically does not change. However, in the transition period (before the stabilization of the motion), there is a significant difference in the trajectories of particle motion, which increases as the angle of inclination of the rectilinear generating surfaces decreases. This gives reason to consider material separation at the stage of the transition period, which requires further theoretical and experimental research.

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ДОСЛІДЖЕННЯ РУХУ МАТЕРІАЛЬНОЇ ЧАСТИНКИ ПО ПОВЕРХНІ КОСОГО ГЕЛІКОЇДА ПІД ДІЄЮ СИЛИ ВЛАСНОЇ ВАГИ

С.Ф. Пилипака, А. В. Несвідомін

Анотація. *Задача, яка описує рух частинки по гвинтовій поверхні, знайшла застосування при проектування спіральних сепараторів. Моделювання руху матеріальної частинки по гвинтових поверхнях і його дослідження сучасними методами чисельного інтегрування і візуалізації дає можливість одержати реальну картину руху за відсутності натурних моделей таких поверхонь. Це дає можливість вести пошук гвинтових поверхонь на предмет покращення їх експлуатаційних характеристик.*

Метою дослідження було встановлення закономірності руху матеріальної частинки по косому гелікоїду в залежності від конструктивних параметрів поверхні.

Конструктивних параметрів у косого гелікоїда два – крок h *і кут* β *нахилу прямолінійних твірних поверхні до горизонтальної площини. За допомогою цих параметрів є більше можливостей впливати на характер руху частинки по косому гелікоїду порівняно з гвинтовим коноїдом і розгортним гелікоїдом.*

Було складено диференціальні рівняння руху матеріальної частинки по поверхні косого гелікоїда під дією сили власної ваги. Рівняння розв’язані чисельними методами. Зроблено візуалізацію одержаних результатів

Встановлено, що рух матеріальної частинки із відомим коефіцієнтом тертя по поверхні косого гелікоїда можна забезпечити на заданій відстані від його осі комбінаціями конструктивних параметрів поверхні. При цьому роздільна здатність поверхні при сепарації частинок із різними коефіцієнтами тертя практично не змінюється. Однак в перехідний період (до стабілізації руху) спостерігається значна різниця у траєкторіях руху частинок, яка зростає по мірі зменшення кута нахилу прямолінійних твірних поверхні. Це дає підстави розглядати сепарацію матеріалу на етапі перехідного періоду, що потребує подальших теоретичних і експериментальних досліджень.

Ключові слова: ***косий гелікоїд, матеріальна частинка, траєкторія руху***