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ON THE SET OF THE FINAL LEVEL CONTINUOUS FUNCTIONS

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This paper considers continuous functions of one real variable with a set of finite levels of the second category.

All further arguments concerning the one-dimensional Euclidean space; also we consider continuous function $f(x)$ is defined on the segment $[a,b]$. We should recall some concepts.

The set A is dense in the set B, if B is contained in the closure of the set A and A is nowhere-dense set if every open set G has another open set $G' \subset G$ containing no points of set A. If a set can be represented as a countable union of nowhere-dense sets, it is called the set of the first category. Supplement to the set of the first category is the set of second category.

A closed nowhere-dense set is zero-dimensional. The set $f^{-1}f(x_0),\ x_0 \in [a,b]$ will be called a level of function $f(x)$. Mapping is zero-dimensional, if each of its level – zero-dimensional set.

Finally, function $n(y)$ on segment $[m,M]$ is called Banachindicatrix, if $m = \min\limits_{a \le x \le b} f(x),\ M = \max\limits_{a \le x \le b} f(x)$ and its values – the number of roots of the equation $f(x) = y$.

Knowledge of many properties of the set of all levels of function (mapping) often makes it possible to characterize its structural features. For continuous mapping its levels are closed sets of the segment, finite or infinite. Furthermore, the examples show that there exist continuous mappings, in which all levels not only infinit but also uncountable. There are mappings in which all levels even are perfect, that does not contain isolated points.

Thereby, an important problem is to study in terms of category the set of levels of real functions of one real variable.

Particularly important in many cases were research related to continuous mappings that have not the intervals of constancy. The set of levels of continuous mapping can be characterized by using category approach .

It turns out that if a continuous mapping with a set of finite levels of second category is zero-dimensional, then the set of its infinite (countable or uncountable) levels is nowhere-dense.

As the famous example of Cantor function, additional requirement on the zero-dimensionality of mapping is essential here. This function is constant on the closure of each of the complementary interval to Cantor set in the segment, but it is increasing in dense open set (the union of these intervals) and the set of its values is everywhere- dense (of the first category only).

Said result can be reformulated as follows. For continuous zero-dimensional mapping the set of its finite level, which is the second category, contains interior points relative to the real line in any interval in the image.

So, if a continuous function is nowhere-constancy on the segment and the set of its finite levels of the second category, this set contains interior points and in an eighbourhood of each such point Banachindicatrix limited. In other words, for a continuous and nowhere-constancy function on the segment, the set of finite levels with precision to a nowhere-dense set can be considered as open and everywhere-dense, if it is the second category.